T.C.
ANKARA YILDIRIM BEYAZIT UNIVERSITY
THE INSTITUTE OF SOCIAL SCIENCES

SMOOTH STRUCTURAL CHANGES AND COINTEGRATION ANALYSIS IN PANEL DATA

Ph.D. DISSERTATION

ÇAĞIN KARUL

THE DEPARTMENT OF ECONOMICS

ASSOC. PROF. DR.
FATİH CEMİL ÖZBUĞDAY
SUPERVISOR

PROF. DR.
ŞABAN NAZLIOĞLU
CO-SUPERVISOR

ANKARA, 2023

APPROVAL PAGE

The thesis prepared by Çağın KARUL titled "Smooth Structural Changes and Cointegration Analysis in Panel Data" has been accepted by the jury members as a thesis at Ankara Yıldırım Beyazıt University, Graduate School of Social Sciences, Department of Economics.

Committee Members		Institution	Signature
Prof. Dr. Hüseyin AĞIR		Ankara Hacı Bayram Veli University	
Accept [X]	Reject [ ]		
Prof. Dr. İlhan KÜÇÜKKAPLAN		Pamukkale University	
Accept [X]	Reject [ ]		
Assoc. Prof. Dr. Fatih Cemil ÖZBUĞDAY		Ankara Yıldırım Beyazıt University	
Accept [X]	Reject [ ]		
Assoc. Prof. Dr. Önder ÖZGÜR		Ankara Yıldırım Beyazıt University	
Accept [X]	Reject [ ]		
Asst. Prof. Dr. Koray GÖKSAL		Ankara Yıldırım Beyazıt University	
Accept [X]	Reject [ ]		

Thesis Defense Date: 09.02.2023

I certify that this thesis fulfills all the requirements to be deemed a Ph. D. dissertation at Ankara Yıldırım Beyazıt University, Graduate School of Social Sciences, Department of Economics.

Director of the Institute of Social Sciences Prof. Dr. Yaşar YİĞİT

……

PLAGIARISM PAGE

I hereby declare that all information in this thesis has been obtained and presented by academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work; otherwise I accept all legal responsibility. 09.02.2023

Çağın KARUL

To Güzin and Atlas

ACKNOWLEDGEMENTS

I will always be indebted to a number of people for their generous support, assistance and encouragement throughout the course of this study. I sincerely wish to acknowledge the following who have helped in bringing this research to fruition, though this is scant thanks for their contributions.

First of all, I would like to thank my supervisors, Assoc. Prof. Dr. Fatih Cemil ÖZBUĞDAY and Prof. Dr. Şaban NAZLIOĞLU for the tremendous amount of support, encouragement and expertise they have offered me throughout this dissertation.

I also sincerely thank to examining committee members, Prof. Dr. Hüseyin AĞIR, Prof. Dr. İlhan KÜÇÜKKAPLAN, Assoc. Prof. Dr. Önder ÖZGÜR and Asst. Prof. Dr. Koray GÖKSAL for their time, invaluable and insightful comments, suggestions and advices.

I extend my special thanks to Dr. Mert AKYÜZ, Dr. Ahmet KONCAK, Dr. Erdal BERK, Recep TAŞKIN and Emre DESTAN for their support during the period.

My gratitude to my parents, Atiye KARUL and Erol KARUL, is beyond the expression with words, for their entirely unreserved love and support throughout my life.

My deepest appreciation belongs to my wife Güzin DEMİRÖZ KARUL who has been walking by my side with her all love, and has been supporting, motivating, and guiding me in every part of my life. Last but not the least, I would appreciate my son Atlas KARUL, who have made me stronger, better and more fulfilled.

ABSTRACT

Smooth Structural Changes and Cointegration Analysis in Panel Data

This dissertation consists of three essays on testing cointegration with structural breaks in time series and panel data. The first study suggests a simple cointegration test with smooth structural changes for the null of no cointegration. The test is based on the Lagrange Multiplier approach, and it can admit serially correlated and heteroskedastic errors, deterministic trends, and smooth structural breaks. The asymptotic distribution of proposed test statistic is derived, and the asymptotic results show that the distribution is free of nuisance parameter dependencies, other than the Fourier frequency. Moreover, the asymptotic analysis reveals that the new test is independent of the number of regressors. To make the testing procedure relatively simple and easy to implement, we use the response surface functions to obtain finite-sample critical values and corresponding p-values. We perform Monte Carlo simulations to evaluate the finite-sample size and power of the suggested test. The simulation results reveal that the proposed test is good size and power properties under different types of structural breaks.

The second study proposes new Lagrange Multiplier-based cointegration tests with structural breaks for dependent panels. To take into account unknown multiple structural breaks, we utilize the Fourier approach that can capture unknown multiple structural changes by using small number of frequencies. On the other hand, we use the PANIC approach to consider cross-section dependency. We derive the asymptotic distribution of suggested test statistic and find that the distribution of test statistics only depends on the Fourier frequency and independent of the number of regressors and common factors. The Monte Carlo simulations results reveal that proposed tests are good size and power properties for stationary or non-stationary common factors under the different types of structural breaks.

The third study examines the validity of PPP hypothesis for 17 OECD countries by employing our new panel cointegration tests with smooth breaks. The purpose of this study is to show the practical application of the new tests and to compare the results with the panel cointegration test with sharp breaks. The results from our suggested cointegration tests provide evidence in favor of PPP validity which is consistent with theoretical expectations.

Keywords: cointegration, Fourier, cross-section dependency, PANIC, purchasing power parity

ÖZET

Yumuşak Yapısal Kırılmalar ve Panel Veri'de Eşbütünleşme Analizi

Bu tez, zaman serileri ve panel veri alanında eşbütünleşmenin yapısal kırılmalar altında test edilmesine dayanan üç makaleden oluşmaktadır. İlk çalışma, eşbütünleşme yoktur boş hipotezi için yumuşak kırılmaları dikkate alan basit bir eşbütünleşme testi önermektedir. Bu test, Lagrange Çarpanı yöntemine dayanmaktadır ve otokorelasyona ve değişen varyansa sahip hatalara, deterministik trende ve yumuşak yapısal kırılmalara izin vermektedir. Önerilen testin asimptotik dağılımı türetilmiş ve sonuçlar dağılımın sadece Fourier frekansından etkilendiğini göstermiştir. Ayrıca asimptotik analiz, testin açıklayıcı değişken sayından da etkilenmediğini göstermiştir. Test sürecini ve uygulamasını kolaylaştırmak amacıyla küçük örneklem kritik değerlerini ve ilgili olasılık değerlerini elde etmek için tepki yüzey fonksiyonları kullanılmıştır. Önerilen testin küçük örneklem özellikleri, Monte Carlo simülasyonlarıyla elde edilmiştir. Simülasyonlar, testin farklı yapısal kırılma tipleri altında iyi boyut ve güç özelliklerine sahip olduğunu göstermiştir.

İkinci çalışmada yatay kesit bağımlılığına sahip paneller için yapısal kırılmalar altında Lagrange Çarpanına dayalı yeni eşbütünleşme testleri geliştirilmektedir. Bilinmeyen çok sayıda yapısal kırılmayı dikkate almak için az sayıda frekans kullanarak bilinmeyen çoklu yapısal değişiklikleri yakalayabilen Fourier yaklaşımından yararlanılmıştır. Diğer yandan, yatay kesit bağımlılığını hesaba katmak için PANIC yaklaşımı kullanılmıştır. Geliştirilen testin asimptotik dağılımı türetilmiş ve asimptotik dağılımın sadece Fourier frekansına bağımlı ve açıklayıcı değişken sayından bağımsız olduğu bulunmuştur. Monte Carlo simülasyon sonuçları durağan veya durağan olmayan ortak faktörler için farklı yapısal kırılma tipleri altında iyi boyut ve güç özelliklerine sahip olduğunu göstermiştir.

Son çalışmada, 17 OECD ülkesi için PPP hipotezinin geçerliliği yeni geliştirilen yumuşak yapısal kırılmalı panel eşbütünleşme testleri kullanılarak incelenmiştir. Bu çalışmanın amacı yeni geliştirilen testleri pratikte kullanımını göstermek ve elde edilen sonuçları ani kırılmaları dikkate alan panel eşbütünleşme testi ile karşılaştırmaktır. Yeni önerilen testlerden elde edilen sonuçlar, teorik beklenti ile uyumlu olarak PPP hipotezinin geçerli olduğu yönünde kanıtlar sunmuştur.

Anahtar Kelimeler: eşbütünleşme, Fourier, yatay kesit bağımlılığı, PANIC, satın alma gücü paritesi

TABLE OF CONTENTS

ABSTRACT i

ÖZET ii

TABLE OF CONTENTS iii

LIST OF ABBREVIATIONS v

LIST OF FIGURES vi

LIST OF TABLES vii

1. INTRODUCTION 1

2. AN LM COINTEGRATION TEST WITH SMOOTH BREAKS 4

2.1. Introduction 4

2.2. Model and testing procedure 5

2.3. Asymptotic distribution of test statistic 8

2.4. Response surface estimates 15

2.5. Small sample simulations 19

2.6. Concluding remarks 26

3. TESTING FOR COINTEGRATION IN DEPENDENT PANELS WITH SMOOTH STRUCTURAL BREAKS 27

3.1. Introduction 27

3.2. Model and testing procedure 30

3.3. Asymptotic distribution of test statistic 33

3.3.1. Panel statistics 39

3.3.2. Response surfaces to approximate *p*-values 42

3.4. Small sample simulations 42

3.5. Concluding remarks 53

4. RE-EXAMINING THE PURCHASING POWER PARITY HYPOTHESIS FOR OECD COUNTRIES 54

4.1. Introduction 54

4.2. Methodology 55
4.3. Data and Empirical results 58
4.3.1. Westerlund and Edgerton (2008) Data 58
4.3.2. Updated Data 61
4.4. Conclusion 67
5. CONCLUSION 68
6. REFERENCES 70
7. APPENDICES 77
7.1. Appendix A 77
7.2. Appendix B 95

LIST OF ABBREVIATIONS

OECD	: Organisation for Economic Co-operation and Development
LM	: Lagrange Multiplier
PANIC	: Panel Analysis of Non-Stationarity in Idiosyncratic and Common Components
PPP	: Purchasing Power Parity
ADL	: Autoregressive Distributive Lag
DGP	: Data Generating Process
MA	: Moving Average
AR	: Autoregressive
LSTAR	: Logistic Smooth Transition Autoregressive
ESTAR	: Exponential Smooth Transition Autoregressive
CA	: Cross-Section Average
DOLS	: Dynamic Ordinary Least Squares
OLS	: Ordinary Least Squares
AIC	: Akaike Information Criterion
BIC	: Schwarz Information Criterion
IC	: Information Criterion
WE	: Westerlund and Edgerton
LOP	: Law of One Price
MSB	: Modified Sargan-Bhargava
IFS	: International Financial Statistics
IMF	: International Monetary Fund
EU	: European Union

LIST OF FIGURES

Figure 2. 1. Illustrations of Sharp, LSTAR, ESTAR, and Fourier breaks in the DGPs 21
Figure 4. 1. The logarithm of nominal exchange rates with sharp (dotted line) and smooth (dashed line) breaks 59
Figure 4. 2. The logarithm of relative prices with sharp (dotted line) and smooth (dashed line) breaks 60
Figure 4. 3. The logarithm of nominal exchange rates with sharp (dotted line) and smooth (dashed line) breaks 63
Figure 4. 4. The logarithm of relative prices with sharp (dotted line) and smooth (dashed line) breaks 64

LIST OF TABLES

Table 2.1. Response surface estimates with no lags for the τFLM statistic with cumulative Fourier frequencies 16

Table 2.2. Response surface estimates with no lags for the τFLM statistic with a single Fourier frequency 17

Table 2.3. Response Surface Estimates with lags for the τFLM statistic with cumulative Fourier frequencies 18

Table 2.4. Response Surface Estimates with lags for the τFLM statistic with a single Fourier frequency 19

Table 2.5. Size properties of the LM-based cointegration test statistics 22

Table 2.6. Power properties of the LM-based cointegration test statistics 24

Table 3.1. Asymptotic moments 39

Table 3.2. Size properties of panel statistics for stationary common factors $\psi = 0.9$ 45

Table 3.3. Power properties of panel statistics for stationary common factors $\psi = 0.9$ 47

Table 3.4. Size properties of panel statistics for non-stationary common factors $\psi = 1$ ... 49

Table 3.5. Power properties of panel statistics for non-stationary common factors $\psi = 1$ 51

Table 4.1: Results from panel unit root tests for Westerlund and Edgerton (2008) data ... 61

Table 4.2: Results from panel cointegration tests for Westerlund and Edgerton (2008) data 62

Table 4.3: Results from Panel unit root tests for updated data 65

Table 4.4: Results from panel cointegration tests for updated data 66

Table 7.1. Computed critical values using the response surface estimates for cumulative Fourier frequencies 77

Table 7.2. Computed critical values using the response surface estimates for single Fourier frequency 78

Table 7.3. Size properties of the LM-based cointegration test statistics with $h = 2$ 79

Table 7.4. Power properties of the LM-based cointegration test statistics with $h = 2$ 81

Table 7.5. Size properties of the LM-based cointegration test statistics with $h = 3$ 83

Table 7.6. Power properties of the LM-based cointegration test statistics with $h = 3$ 85

Table 7.7. Size properties of the LM-based cointegration test statistics with $h = 4$ 87

Table 7.8. Power properties of the LM-based cointegration test statistics with $h = 4$ 89

Table 7.9. Size properties of the LM-based cointegration test statistics with $h = 5$ 91

Table 7.10. Power properties of the LM-based cointegration test statistics with $h = 5$ 93
Table 7.11. Response surface estimates with no lag for the τFLM statistic with cumulative Fourier frequencies 95
Table 7.12. Response surface estimates with lags for the τFLM statistic with cumulative Fourier frequencies 95
Table 7.13. Size properties of panel statistics with $h = 2$ 96
Table 7.14. Power properties of panel statistics with $h = 2$ 98
Table 7.15. Size properties of panel statistics with $h = 3$ 100
Table 7.16. Power properties of panel statistics with $h = 3$ 102
Table 7.17. Size properties of panel statistics with $h = 4$ 104
Table 7.18. Power properties of panel statistics with $h = 4$ 106
Table 7.19. Size properties of panel statistics with $h = 5$ 108
Table 7.20. Power properties of panel statistics with $h = 5$ 110
Table 7.21. Size properties of panel statistics with $rmax = 2$ 112
Table 7.22. Power properties of panel statistics with $rmax = 2$ 114
Table 7.23. Size properties of panel statistics with $rmax = 3$ 116
Table 7.24. Power properties of panel statistics with $rmax = 3$ 118
Table 7.25. Size properties of panel statistics with $rmax = 4$ 120
Table 7.26. Power properties of panel statistics with $rmax = 4$ 122
Table 7.27. Size properties of panel statistics with $rmax = 6$ 124
Table 7.28. Power properties of panel statistics with $rmax = 6$ 126
Table 7.29. Size properties of panel statistics with $rmax = 7$ 128
Table 7.30. Power properties of panel statistics with $rmax = 7$ 130

1. INTRODUCTION

The notion of cointegration was first presented by Granger (1981) and elaborated by Engle and Granger (1987). For a long-run relationship to exist, a linear combination of two or more non-stationary variables should be stationary. Hence, these variables are said to be cointegrated (Asteriou and Hall, 2007, p. 307). More generally, suppose that we consider two variables y_t and x_t that are integrated of order d, i.e. $y_t, x_t \sim I(d)$, then y_t and x_t are cointegrated if there exists a β such that $y_t - \beta x_t$ is integrated of a smaller order than d (Maddala and Kim, 1998, p. 26).

It is known that if some or all variables in a regression are $I(1)$ and not cointegrated, then the spurious regression problem arises (Granger and Newbold, 1974). Phillips (1986) shows the behavior of the slope parameter in the spurious regression. According to results of this paper, the slope parameter does not convergence in probability to zero but instead converges in distribution to a non-normal random variable. Phillips (1986) also show that the usual t-statistics for testing that the elements of the slope parameter are zero diverge to $\pm\infty$ as $T \to \infty$ and R^2 converges to unity as $T \to \infty$ so that the model will appear to fit well even though it is misspecified. Thus, regression with $I(1)$ data only makes sense when the data are cointegrated. This result reveals the importance of determining the cointegration relationship between variables.

This study focuses on testing the cointegration in time series and panel data analysis frameworks and suggests new cointegration tests for both contexts. Moreover, we also re-examining the purchasing power parity hypothesis for OECD (Organisation for Economic Co-operation and Development) countries to demonstrate the practical application of the new tests.

This dissertation consists of three essays on testing cointegration with structural breaks in time series and panel data in the following three chapters. The next chapter proposes a simple test for the null of no cointegration that allow structural changes. The test is derived

from the Lagrange Multiplier (LM)-based unit root tests developed by Schmidt and Phillips (1992) and Enders and Lee (2012), and it can admit serially correlated and heteroskedastic errors, deterministic trends, and smooth structural breaks. We derive the asymptotic distribution of proposed test statistic and find that it is free of nuisance parameter dependencies, other than the Fourier frequency. Moreover, the asymptotic analysis reveals that the new test is independent of the number of regressors. We present the convenient response surface estimates to obtain finite-sample critical values and corresponding p-values, making the testing procedure relatively simple and easy to implement. We use Monte Carlo simulations to evaluate the finite-sample size and power of the suggested test. The simulation results show that the proposed test is good size and power properties under different types of structural breaks.

The recent panel cointegration literature concentrate on the existence of cross-sectional dependency and the structural breaks. Although Hao (1996) shows that neglecting structural breaks lead to size distortions and deceptive inference when testing for cointegration, there are only a few papers that consider cross-section dependency and structural breaks (Westerlend, 2006; Westerlund and Edgerton, 2008; Banerjee and Carrion-i-Silvestre, 2015; Olayeni et al., 2021) in the literature. To fill this gap, the third chapter develops new Lagrange Multiplier (LM)-based cointegration tests with structural breaks for dependent panels. We apply the Fourier approach to capture multiple structural breaks. This approach can capture unknown multiple structural changes by using small number of frequencies (Nazlioglu et al., 2022). Moreover, we utilize the PANIC (panel analysis of non-stationarity in idiosyncratic and common components) approach of Bai and Ng (2004) to consider cross-section dependency. The asymptotic results of our tests indicate that the distribution of test statistics only depends on the Fourier frequency and independent of the number of regressors and common factors. We provide a response surface function to compute the critical values of the univariate LM cointegration test statistics and the corresponding p-value for finite samples. Finally, we perform Monte Carlo simulations to evaluate the finite-sample size and power of the suggested tests. The simulation results reveal that proposed tests are good size and power properties for stationary or non-stationary common factors under the different types of structural breaks.

The fourth chapter examines the validity of Purchasing Power Parity (PPP) hypothesis for 17 OECD countries by employing our new panel cointegration tests that simultaneously

address the issues of structural breaks, cross-sectional dependence, and heterogeneity at the same time. The main purpose of this analysis is to show the practical application of the new tests. To this end, we utilize two datasets to testing of PPP validity. We use Westerlund and Edgerton (2008) dataset for the purpose of comparison and use the updated dataset to investigate the current validity of PPP. The results from our suggested cointegration tests provide evidence in favor of PPP validity which is consistent with theoretical expectations.

2. AN LM COINTEGRATION TEST WITH SMOOTH BREAKS

2.1. Introduction

Considering structural breaks in cointegration analysis has recently attracted great attention in both empirical and theoretical literature. The empirical reason for this interest is that many macroeconomic time series, such as unemployment rate, wages, gross national product, production, money stock, employment and interest rates contain structural breaks, as shown in Perron (1989) and Stock and Watson (1996). On the other hand, the theoretical reason for this interest is that, as discussed in Leybourne and Newbold (2003) and Cook (2004), ignoring structural breaks in a cointegration analysis leads to false conclusions regarding the existence of a cointegration relationship between the variables.

In order to overcome this spurious rejection problem, pioneering studies use the dummy variable approach to accommodate structural breaks in the data generating process. First, studies consider one sharp break in the data (Gregory and Hansen, 1996; Inoue, 1999; Lütkepohl et al., 2004; Westerlund and Edgerton, 2007b; Arai and Kurozumi, 2007) and then studies that take into account two or more sharp breaks are developed (Hatemi-J, 2008; Maki, 2012). However, the problem here is that as the number of dummy variables used to capture multiple structural breaks increases, the number of parameters to be estimated increases, too, and the power of the test decreases significantly. Therefore, most recent papers in testing cointegration with structural breaks literature have focused on smooth breaks. Gallant (1981) and Gallant and Souza (1991) suggest the Fourier approximation to capture the unknown multiple structural breaks. This approach performs best with smooth breaks but can also accommodate sharp breaks in the data (Banerjee et al. 2017). Moreover, the Fourier approximation does not require information about the functional form of breaks, the number of breaks, and estimating break dates. Based on this idea, Tsong et al. (2016) propose the residual-based test for the null of cointegration and Banerjee et al. (2017) develop Autoregressive Distributive Lag (ADL) cointegration test for the null of no cointegration. The distributions of these tests' statistics depend on the number of regressors. As shown in Westerlund and Edgerton (2007b), LM-based cointegration tests are unaffected by the regressors. To the best of our knowledge, there is no LM-based cointegration test with Fourier approximation in the time-series cointegration literature.

This study proposes a simple test for the null of no cointegration that allows structural changes. The test is derived from the LM-based unit root tests developed by Schmidt and Phillips (1992) and Enders and Lee (2012), and it can admit serially correlated and heteroskedastic errors, deterministic trends, and smooth structural breaks. We derive the asymptotic distribution of the proposed test statistic and find that it is free of nuisance parameter dependencies, other than the Fourier frequency. Moreover, the asymptotic analysis reveals that the new test is independent of the number of regressors. Therefore, we do not need to compute the critical values, since the distribution of test statistic is identical to that obtained by Enders and Lee (2012). While the critical values are already available in Table 1 in Enders and Lee (2012, p. 580), we present the convenient response surface estimates to obtain finite-sample critical values and corresponding p-values, making the testing procedure relatively simple and easy to implement. Finally, we perform Monte Carlo simulations to evaluate the finite-sample size and power of the suggested test. The simulation results reveal that the proposed test has good size and power properties under different types of structural breaks.

The rest of the paper is as follows. The next section describes the model and testing procedure. Section 3 illustrates the asymptotic distribution of the proposed test statistic and the response surfaces to approximate p-values. Section 4 is devoted to small sample analysis. The concluding remarks are finally presented in section 5.

2.2. Model and testing procedure

The data generating process (DGP) can be written as follows:

$$y_t = d(t) + x_t'\beta + e_t \qquad t = 1, \dots T \tag{2.1}$$

$$x_t = x_{t-1} + w_t \tag{2.2}$$

$$e_t = \rho e_{t-1} + \varepsilon_t \tag{2.3}$$

where y_t is the dependent variable, x_t is an h-dimensional vector of regressors, $d(t)$ represents the deterministic terms, and T shows the number of observations. The initial conditions x_0 and e_0 are treated as fixed for convenience in deriving the asymptotic distribution of the test statistic. We make the following assumptions for the error processes ε_t and w_t.

Assumption 1: The processes ε_t and ω_t satisfy the following set of conditions:

i. $E(\varepsilon_t w_j) = 0$ for all *t* and *j*;

ii. The process ε_t is normal, independent, and identically distributed (i.i.d.) with $E(\varepsilon_t) = 0$ and $E(\varepsilon_t^2) = \sigma_\varepsilon^2 > 0$;

iii. $E(w_t) = 0$ and $E(w_t w_t') = \Omega$ is positive definite.

Assumption 1 embodies the usual assumptions for the new cointegration tests. Assumption 1(i) and (iii), are related to the joint covariance matrix of ε_t and w_t. If the regressors are exogenous, Assumption 1(i) indicates that ε_t and w_t are uncorrelated. Although this assumption is quite restrictive, it will be relaxed later on[1]. Assumption 1(iii) emphasizes that the hxh covariance matrix Ω is positive definite, which is equivalent to requiring that x_t is not cointegrated in case we have multiple regressors. Assumption 1(ii) is very standard for the likelihood-based framework, and it states that the innovations ε_t are i.i.d. and normal. It allows the use of standard functional central limit arguments, which makes the asymptotic analysis relatively uncomplicated. Furthermore, it is possible to modify our test to allow for heteroskedasticity and serial correlation in the error term.

An important issue is capturing possible breaks in the deterministic term $d(t)$. To this end, we use Fourier approximation which is based on a variant of the Flexible Fourier Form by Gallant (1981) for considering structural breaks. The Fourier approximation does not require prior knowledge of the number, dates, and form of breaks and captures structural breaks as a gradual/smooth process by using a small number of low-frequency components. Following Becker et al. (2006) and Enders and Lee (2012), the Fourier function with an unknown form of breaks can be described as follows:

$$d(t) = \alpha_0 + \alpha_1 t + \sum_{k=1}^{m} \gamma_{1k} sin\left(\frac{2\pi kt}{T}\right) + \sum_{k=1}^{m} \gamma_{2k} cos\left(\frac{2\pi kt}{T}\right); \qquad m < \frac{T}{2} \tag{2.4}$$

where *m* is the number of frequencies, γ_{1k} and γ_{2k} respectively measure the amplitude and displacement. As shown in Becker et al. (2006), a single Fourier frequency can mimic a variety of breaks in deterministic components regardless of date, number, and form of breaks. Hence, it can be defined as

$$d(t) = \alpha_0 + \alpha_1 t + \gamma_1 \sin\left(\frac{2\pi kt}{T}\right) + \gamma_2 \cos\left(\frac{2\pi kt}{T}\right) \tag{2.5}$$

[1] See Remark 2.2.

where k denotes the frequency for the approximation. It is clear from equation (2.5) that if $\gamma_{1i} = \gamma_{2i} = 0$ the model reduces to the linear specification, and the cointegration procedure with no structural break proposed by Westerlund and Edgerton (2007b) is appropriate.

After defining the key assumptions and the deterministic terms, the null of no cointegration can be tested using equations (2.1) – (2.3). When x_t and y_t are cointegrated, the error e_t is stationary but if they are not, then it has a unit root. Therefore, the null hypothesis of no cointegration ($H_0: \rho = 1$) is tested against the alternative hypothesis of cointegration ($H_1: \rho < 1$). To this end, we use the Lagrange Multiplier (LM) methodology of Schmidt and Philips (1992) and Enders and Lee (2012). By imposing the null restriction, we estimate the following ordinary least square (OLS) regression using the first differences:

$$\Delta y_t = \delta_0 + \delta_1 \Delta \sin\left(\frac{2\pi k t}{T}\right) + \delta_2 \Delta \cos\left(\frac{2\pi k t}{T}\right) + \Delta x_t' \beta + u_t \qquad (2.6)$$

where $u_t = \Delta e_t$. Then, using these estimated coefficients ($\tilde{\delta}_0$, $\tilde{\delta}_1$, $\tilde{\delta}_2$, and $\tilde{\beta}$), we construct a detrended series as:

$$\tilde{S}_t = y_t - \tilde{\psi} - \tilde{\delta}_0 t - \tilde{\delta}_1 \sin\left(\frac{2\pi k t}{T}\right) - \tilde{\delta}_2 \cos\left(\frac{2\pi k t}{T}\right) - x_t' \tilde{\beta}, \quad t = 2, \dots, T \qquad (2.7)$$

where $\tilde{\psi} = y_1 - \tilde{\delta}_0 - \tilde{\delta}_1 \sin\left(\frac{2\pi k}{T}\right) - \tilde{\delta}_2 \cos\left(\frac{2\pi k}{T}\right) - x_1' \tilde{\beta}$, and y_1 and x_1 are first observations of y_t and x_t, respectively. Finally, the testing regression with detrended series can be written as follows:

$$\Delta \tilde{S}_t = \phi \tilde{S}_{t-1} + constant + c_1 \Delta \sin\left(\frac{2\pi k t}{T}\right) + c_2 \Delta \cos\left(\frac{2\pi k t}{T}\right) + error. \qquad (2.8)$$

The null hypothesis of no cointegration versus the alternative of cointegration can be formulated equivalently as $H_0: \phi = 0$ versus $H_1: \phi < 0$. Thus, the LM test statistic is:

$$\tau_{FLM} = \frac{\tilde{\phi}}{SE(\tilde{\phi})} \qquad (2.9)$$

where $\tilde{\phi}$ and $SE(\tilde{\phi})$ are the OLS estimate of ϕ and the standard error of $\tilde{\phi}$ in equation (2.8), respectively.

2.3. Asymptotic distribution of test statistic

In this section, we derive the limiting distribution of the LM-based test statistic under the null hypothesis ($H_0: \phi = 0$). For simplicity, the notation $(x_t)_p$ is to state the error from projecting x_t onto the generic vector X_t of projection variables. Therefore, $(x_t)_p = x_t - X_t'b$, where b is a vector of projection parameters.

Theorem 2.1: Suppose that y_t and x_t are generated by the DGP in equations (2.1) – (2.3) with $\rho = 1$, and one adopts the first step testing regression equation (2.6). Under H_0 and Assumption 2.1, as $T \to \infty$, then,

$$\tau_{FLM} \to -\frac{1}{2}\left(\frac{\sigma_\varepsilon}{\sigma}\right)\left[\int_0^1 \underline{V}(r)^2 dr\right]^{-\frac{1}{2}} \tag{2.10}$$

where $\underline{V}(r)$ is the projection of the process $V(r)$ on the orthogonal complement of the space spanned by the trigonometric function $dz = \left(1, dsin(2\pi kr), dcos(2\pi kr)\right)'$, with $r \in [0,1]$.

Proof: Let us begin by considering the limit of $\tilde{S}_t$. We can write $\tilde{S}_t$ as follows.

$$\tilde{S}_t = y_t - \tilde{\psi} - \tilde{\delta}_0 t - \tilde{\delta}_1 \sin\left(\frac{2\pi kt}{T}\right) - \tilde{\delta}_2 \cos\left(\frac{2\pi kt}{T}\right) - x_t'\tilde{\beta}.$$

By inserting $\tilde{\psi} = y_1 - \tilde{\delta}_0 - \tilde{\delta}_1 \sin\left(\frac{2\pi k}{T}\right) - \tilde{\delta}_2 \cos\left(\frac{2\pi k}{T}\right) - x_1'\tilde{\beta}$, we have

$$\begin{aligned}\tilde{S}_t = y_t - y_1 - \tilde{\delta}_0(t-1) &- \tilde{\delta}_1\left(\sin\left(\frac{2\pi kt}{T}\right) - \sin\left(\frac{2\pi k}{T}\right)\right) \\ &- \tilde{\delta}_2\left(\cos\left(\frac{2\pi kt}{T}\right) - \cos\left(\frac{2\pi k}{T}\right)\right) - (x_t - x_1)'\tilde{\beta} \\ = \sum_{j=2}^{t} \varepsilon_j - (x_t - x_1)'(\tilde{\beta} - \beta) &- (\tilde{\delta}_0 - \delta_0)(t-1) \\ &- (\tilde{\delta}_1 - \delta_1)\left(\sin\left(\frac{2\pi kt}{T}\right) - \sin\left(\frac{2\pi k}{T}\right)\right) \\ &- (\tilde{\delta}_2 - \delta_2)\left(\cos\left(\frac{2\pi kt}{T}\right) - \cos\left(\frac{2\pi k}{T}\right)\right). \end{aligned} \tag{2.11}$$

Let $\Delta y, \Delta x$ and ε be the sample means of $\Delta y_t, \Delta x_t$ and ε_t, respectively. Note that the sample means of $\Delta\sin\left(\frac{2\pi kt}{T}\right)$ and $\Delta\cos\left(\frac{2\pi kt}{T}\right)$ are zero since $\sum_{t=2}^{T}\Delta\sin\left(\frac{2\pi kt}{T}\right) = \sum_{t=2}^{T}\Delta\cos\left(\frac{2\pi kt}{T}\right) = 0$. Furthermore, let w be the mean of w_t. Then, the third term on the right-hand side of the above equation can be written as

$$\begin{aligned}\left(\tilde{\delta}_0 - \delta_0\right) &= \Delta y - \Delta x'\tilde{\beta} - \delta_0 \\ &= \varepsilon - \Delta x'\left(\tilde{\beta} - \beta\right) \\ &= \varepsilon - \mathrm{w}'\left(\tilde{\beta} - \beta\right).\end{aligned}$$

Hence, by using equation (2.11) we get

$$\begin{aligned}\frac{1}{\sqrt{T}}\tilde{S}_t = \frac{1}{\sqrt{T}}\sum_{j=2}^{t}\varepsilon_j - \frac{1}{\sqrt{T}}\varepsilon\,(t-1) - \frac{1}{\sqrt{T}}\sum_{j=2}^{t}w_j'\left(\tilde{\beta} - \beta\right) + \frac{1}{\sqrt{T}}\mathrm{w}'\left(\tilde{\beta} - \beta\right)(t-1) \\ - \frac{1}{\sqrt{T}}\left(\tilde{\delta}_1 - \delta_1\right)\left(\sin\left(\frac{2\pi kt}{T}\right) - \sin\left(\frac{2\pi k}{T}\right)\right) \\ - \frac{1}{\sqrt{T}}\left(\tilde{\delta}_2 - \delta_2\right)\left(\cos\left(\frac{2\pi kt}{T}\right) - \cos\left(\frac{2\pi k}{T}\right)\right) \\ = \frac{1}{\sqrt{T}}\sum_{j=2}^{t}\left(\varepsilon_j - \varepsilon\right) - \frac{1}{\sqrt{T}}\sum_{j=2}^{t}\left(w_j - w\right)'\left(\tilde{\beta} - \beta\right) \\ - \frac{1}{\sqrt{T}}\left(\tilde{\delta}_1 - \delta_1\right)\left(\sin\left(\frac{2\pi kt}{T}\right) - \sin\left(\frac{2\pi k}{T}\right)\right) \\ - \frac{1}{\sqrt{T}}\left(\tilde{\delta}_2 - \delta_2\right)\left(\cos\left(\frac{2\pi kt}{T}\right) - \cos\left(\frac{2\pi k}{T}\right)\right) \\ = I - II - III - IV, \text{ say.}\end{aligned}$$

For part *I*, we have $T^{-\frac{1}{2}}\sum_{j=2}^{t}\left(\varepsilon_j - \varepsilon\right) \Rightarrow \sigma V(r)$ as $T \to \infty$, which implies that $I = O_p(1)$. For part *II*, $\frac{1}{\sqrt{T}}\sum_{j=2}^{t}\left(w_j - w\right)' \Rightarrow \sigma\tilde{V}(r)$ as $T \to \infty$, where $\tilde{V}(r)$ is a standard vector Brownian bridge process. By using Assumption 1(i), we have $T^{-\frac{1}{2}}\sum_{t=2}^{T}(\Delta x_t)_p(\varepsilon_t)_p = O_p(1)$ since ε_t and w_t are orthogonal. Moreover, $T^{-\frac{1}{2}}\sum_{t=2}^{T}(\Delta x_t)_p(\Delta x_t)_p' \to_p \Omega$ based on Assumption 1(iii), where $\to_p$ signifies convergence in probability. Hence, $\sqrt{T}\left(\tilde{\beta} - \beta\right) = O_p(1)$. We get

$II = T^{-1}\sum_{j=2}^{t}\left(w_j - w\right)'\sqrt{T}\left(\tilde{\beta} - \beta\right) = O_p\left(T^{-\frac{1}{2}}\right)\cdot O_p(1).$

For part *III*, let $X_t = \left(1, \Delta x_t', \Delta \cos\left(\frac{2\pi kt}{T}\right)\right)'$, then term $\frac{1}{\sqrt{T}}\left(\tilde{\delta}_1 - \delta_1\right)$ can be written as

$$\frac{1}{\sqrt{T}}\left(\tilde{\delta}_1 - \delta_1\right) = \left(T\sum_{t=2}^{T}\left(\Delta \sin\left(\frac{2\pi kt}{T}\right)\right)_p^2\right)^{-1}\sqrt{T}\sum_{t=2}^{T}\left(\Delta \sin\left(\frac{2\pi kt}{T}\right)\right)_p (\varepsilon_t)_p. \qquad (2.12)$$

The denominator of this expression can be expanded as

$$T\sum_{t=2}^{T}\left(\Delta \sin\left(\frac{2\pi kt}{T}\right)\right)_p^2 = T\sum_{t=2}^{T}\left(\Delta \sin\left(\frac{2\pi kt}{T}\right)\right)^2 - \sum_{t=2}^{T}\Delta \sin\left(\frac{2\pi kt}{T}\right)X_t'$$
$$\times\left(T^{-1}\sum_{t=2}^{T}X_t X_t'\right)^{-1}\sum_{t=2}^{T}X_t'\Delta \sin\left(\frac{2\pi kt}{T}\right).$$

We now evaluate each of the terms in this expression.

(a) Since $\Delta \sin\left(\frac{2\pi kt}{T}\right) = \left(\frac{2\pi k}{T}\right)\cos\left(\frac{2\pi kt}{T}\right) + o_p(1)$, the first term follows:

$$T\sum_{t=2}^{T}\Delta \sin^2\left(\frac{2\pi kt}{T}\right) \to (2\pi k)^2\int_0^1 \cos^2(2\pi kr)dr\,.$$

(b) For the numerator of the second term, we have

$$\sum_{t=2}^{T}\Delta \sin\left(\frac{2\pi kt}{T}\right)X_t' = \begin{bmatrix} \sum_{t=2}^{T}\Delta \sin\left(\frac{2\pi kt}{T}\right) \\ \sum_{t=2}^{T}\Delta \sin\left(\frac{2\pi kt}{T}\right)w_t \\ \sum_{t=2}^{T}\Delta \sin\left(\frac{2\pi kt}{T}\right)\Delta \cos\left(\frac{2\pi kt}{T}\right) \end{bmatrix} = \begin{bmatrix} 0 \\ o_p(1) \\ 0 \end{bmatrix}.$$

(c) For the denominator of the second term, we have

$$T^{-1}\sum_{t=2}^{T}X_t X_t' = T^{-1}\sum_{t=2}^{T}\begin{bmatrix} 1 & \Delta x_t & \Delta \cos\left(\frac{2\pi kt}{T}\right) \\ \Delta x_t & \Delta x_t \Delta x_t' & \Delta x_t \Delta \cos\left(\frac{2\pi kt}{T}\right) \\ \Delta \cos\left(\frac{2\pi kt}{T}\right) & \Delta \cos\left(\frac{2\pi kt}{T}\right)\Delta x_t' & \Delta \cos\left(\frac{2\pi kt}{T}\right)^2 \end{bmatrix}.$$

- $T^{-1}\sum_{t=2}^{T}\Delta x_t \to_p 0$, since Δx_t is mean zero by Assumption 1(iii).
- $T^{-1}\sum_{t=2}^{T}\Delta \cos\left(\frac{2\pi kt}{T}\right) = 0$ and $T^{-1}\sum_{t=2}^{T}\Delta \cos\left(\frac{2\pi kt}{T}\right)\Delta x_t' = o_p(1)$.
- $T^{-1}\sum_{t=2}^{T}\Delta x_t \Delta x_t' \to \Omega$ and since $\Delta \cos\left(\frac{2\pi kt}{T}\right) = -\left(\frac{2\pi k}{T}\right)\sin\left(\frac{2\pi kt}{T}\right) + o_p(1)$, $\Delta \cos\left(\frac{2\pi kt}{T}\right)^2$ must be $o_p(1)$ when normalized T^{-1}.

The numerator of equation (2.12) can be written as

$$\sqrt{T}\sum_{t=2}^{T}\left(\Delta\sin\left(\frac{2\pi kt}{T}\right)\right)_p(\varepsilon_t)_p = \sqrt{T}\sum_{t=2}^{T}\Delta\sin\left(\frac{2\pi kt}{T}\right)\varepsilon_t - \sum_{t=2}^{T}\Delta\sin\left(\frac{2\pi kt}{T}\right)X_t' \times\left(T^{-1}\sum_{t=2}^{T}X_{\mathrm{t}}X_t'\right)^{-1}\frac{1}{\sqrt{T}}\sum_{t=2}^{T}X_{\mathrm{t}}'\varepsilon_{\mathrm{t}}.$$

Following Proposition 1 of Enders and Lee (2012, p. 594), the first term follows:

$$\sqrt{T}\sum_{t=2}^{T}\Delta\sin\left(\frac{2\pi kt}{T}\right)\varepsilon_t \to \sigma(2\pi k)\left[W(1) + (2\pi k)\int_0^1 \sin(2\pi kr)\,W(r)dr\right].$$

For the second term, $\sum_{t=2}^{T}\Delta\sin\left(\frac{2\pi kt}{T}\right)X_t'$ and $(T^{-1}\sum_{t=2}^{T}X_{\mathrm{t}}X_t')^{-1}$ are derived earlier. Using Assumption 1(i) and (ii), the last part of the second term follows:

$$\frac{1}{\sqrt{T}}\sum_{t=2}^{T}X_{\mathrm{t}}'\varepsilon_{\mathrm{t}} = \begin{bmatrix}\frac{1}{\sqrt{T}}\sum_{t=2}^{T}\varepsilon_{\mathrm{t}}\\ \frac{1}{\sqrt{T}}\sum_{t=2}^{T}w_t\,\varepsilon_{\mathrm{t}}\\ \frac{1}{\sqrt{T}}\sum_{t=2}^{T}\Delta\sin\left(\frac{2\pi kt}{T}\right)\varepsilon_{\mathrm{t}}\end{bmatrix} = \begin{bmatrix}0\\0\\o_p(1)\end{bmatrix}.$$

By using the above results, we can show that

$$\frac{1}{\sqrt{T}}\left(\tilde{\delta}_1 - \delta_1\right) \to \sigma\left[(2\pi k)\int_0^1\cos^2(2\pi kr)\,dr\right]^{-1}\left[W(1) + (2\pi k)\int_0^1\sin(2\pi kr)\,W(r)dr\right].$$

For part *IV*, let $X_t = \left(1, \Delta x_t', \Delta\sin\left(\frac{2\pi kt}{T}\right)\right)'$, then term $\frac{1}{\sqrt{T}}\left(\tilde{\delta}_2 - \delta_2\right)$ can be written as

$$\frac{1}{\sqrt{T}}\left(\tilde{\delta}_2 - \delta_2\right) = \left(T\sum_{t=2}^{T}\left(\Delta\cos\left(\frac{2\pi kt}{T}\right)\right)_p^2\right)^{-1}\sqrt{T}\sum_{t=2}^{T}\left(\Delta\cos\left(\frac{2\pi kt}{T}\right)\right)_p(\varepsilon_t)_p. \qquad (2.13)$$

The denominator of this expression can be expanded as

$$T\sum_{t=2}^{T}\left(\Delta\cos\left(\frac{2\pi kt}{T}\right)\right)_p^2 = T\sum_{t=2}^{T}\left(\Delta\cos\left(\frac{2\pi kt}{T}\right)\right)^2 - \sum_{t=2}^{T}\Delta\cos\left(\frac{2\pi kt}{T}\right)X_t'$$

$$\times \left(T^{-1} \sum_{t=2}^{T} X_t X_t' \right)^{-1} \sum_{t=2}^{T} X_t' \Delta \cos\left(\frac{2\pi kt}{T}\right).$$

We now evaluate each of the terms in this expression.

(d) Since $\Delta \cos\left(\frac{2\pi kt}{T}\right) = -\left(\frac{2\pi k}{T}\right) \sin\left(\frac{2\pi kt}{T}\right) + o_p(1)$, the first term follows:

$$T \sum_{t=2}^{T} \Delta \cos^2\left(\frac{2\pi kt}{T}\right) \to (2\pi k)^2 \int_0^1 \sin^2(2\pi kr) dr .$$

(e) For the numerator of the second term, we have

$$\sum_{t=2}^{T} \Delta \cos\left(\frac{2\pi kt}{T}\right) X_t' = \begin{bmatrix} \sum_{t=2}^{T} \Delta \cos\left(\frac{2\pi kt}{T}\right) \\ \sum_{t=2}^{T} \Delta \cos\left(\frac{2\pi kt}{T}\right) w_t \\ \sum_{t=2}^{T} \Delta \sin\left(\frac{2\pi kt}{T}\right) \Delta \cos\left(\frac{2\pi kt}{T}\right) \end{bmatrix} = \begin{bmatrix} 0 \\ o_p(1) \\ 0 \end{bmatrix}.$$

(f) For the denominator of the second term, we have

$$T^{-1} \sum_{t=2}^{T} X_t X_t' = T^{-1} \sum_{t=2}^{T} \begin{bmatrix} 1 & \Delta x_t & \Delta \sin\left(\frac{2\pi kt}{T}\right) \\ \Delta x_t & \Delta x_t \Delta x_t' & \Delta x_t \Delta \sin\left(\frac{2\pi kt}{T}\right) \\ \Delta \sin\left(\frac{2\pi kt}{T}\right) & \Delta \sin\left(\frac{2\pi kt}{T}\right) \Delta x_t' & \Delta \sin\left(\frac{2\pi kt}{T}\right)^2 \end{bmatrix}.$$

- $T^{-1} \sum_{t=2}^{T} \Delta x_t \to_p 0$, since Δx_t is mean zero by Assumption 1(iii).
- $T^{-1} \sum_{t=2}^{T} \Delta \sin\left(\frac{2\pi kt}{T}\right) = 0$ and $T^{-1} \sum_{t=2}^{T} \Delta \sin\left(\frac{2\pi kt}{T}\right) \Delta x_t' = o_p(1)$.
- $T^{-1} \sum_{t=2}^{T} \Delta x_t \Delta x_t' \to \Omega$ and since $\Delta \sin\left(\frac{2\pi kt}{T}\right) = \left(\frac{2\pi k}{T}\right) \cos\left(\frac{2\pi kt}{T}\right) + o_p(1)$, $\Delta \sin\left(\frac{2\pi kt}{T}\right)^2$ must be $o_p(1)$ when normalized T^{-1}.

The numerator of equation (2.13) can be written as

$$\sqrt{T} \sum_{t=2}^{T} \left(\Delta \cos\left(\frac{2\pi kt}{T}\right) \right)_p (\varepsilon_t)_p = \sqrt{T} \sum_{t=2}^{T} \Delta \cos\left(\frac{2\pi kt}{T}\right) \varepsilon_t - \sum_{t=2}^{T} \Delta \cos\left(\frac{2\pi kt}{T}\right) X_t'$$
$$\times \left(T^{-1} \sum_{t=2}^{T} X_t X_t' \right)^{-1} \frac{1}{\sqrt{T}} \sum_{t=2}^{T} X_t' \varepsilon_t.$$

Following Proposition 1 of Enders and Lee (2012, p. 594), the first term follows:

$$\sqrt{T}\sum_{t=2}^{T}\Delta\cos\left(\frac{2\pi kt}{T}\right)\varepsilon_t \rightarrow \sigma(2\pi k)^2\left[\int_0^1 \cos(2\pi kr)\,W(r)dr\right].$$

For the second term, $\sum_{t=2}^{T}\Delta\sin\left(\frac{2\pi kt}{T}\right)X_t'$ and $(T^{-1}\sum_{t=2}^{T}X_{\mathrm{t}}X_t')^{-1}$ are derived earlier. Using Assumption 1(i) and (ii), the last part of the second term follows:

$$\frac{1}{\sqrt{T}}\sum_{t=2}^{T}X_{\mathrm{t}}'\varepsilon_{\mathrm{t}} = \begin{bmatrix} \frac{1}{\sqrt{T}}\sum_{t=2}^{T}\varepsilon_{\mathrm{t}} \\ \frac{1}{\sqrt{T}}\sum_{t=2}^{T}w_t\,\varepsilon_{\mathrm{t}} \\ \frac{1}{\sqrt{T}}\sum_{t=2}^{T}\Delta\cos\left(\frac{2\pi kt}{T}\right)\varepsilon_{\mathrm{t}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ o_p(1) \end{bmatrix}.$$

By using the above results, we can show that

$$\frac{1}{\sqrt{T}}\left(\tilde{\delta}_2-\delta_2\right) \rightarrow \sigma\left[\int_0^1 \sin^2(2\pi kr)\,dr\right]^{-1}\left[\int_0^1 \cos(2\pi kr)\,W(r)dr\right].$$

Finally, we can show $\frac{1}{\sqrt{T}}\tilde{S}_t \rightarrow \sigma V(r)$ where

$$V(r) = W(r) - rW(1) - \left[(2\pi k)\int_0^1 \cos^2(2\pi kr)\,dr\right]^{-1}$$
$$\cdot\left[W(1)+(2\pi k)\int_0^1 \sin(2\pi kr)\,W(r)dr\right]\cdot\sin(2\pi kr)$$
$$-\left[\int_0^1 \sin^2(2\pi kr)\,dr\right]^{-1}\left[\int_0^1 \cos(2\pi kr)\,W(r)dr\right]\cdot\cos(2\pi kr)$$

and $W(r)$ is a Wiener process on $r \in [0,1]$.

For the second step of the regression in (2.8), we obtain:

$$\tilde{\phi} = \left(\tilde{S}'M_{\Delta Z}\tilde{S}\right)^{-1}\left(\tilde{S}'M_{\Delta Z}\Delta\tilde{S}\right),$$

where $\Delta Z_t = \left(1, \Delta \sin\left(\frac{2\pi kt}{T}\right), \Delta \cos\left(\frac{2\pi kt}{T}\right)\right)'$, $\tilde{S} = (\tilde{S}_1, \dots, \tilde{S}_{T-1})'$, $\Delta Z = (\Delta Z_2, \dots, \Delta Z_T)'$, $\Delta\tilde{S} = (\Delta\tilde{S}_2, \dots, \Delta\tilde{S}_T)$ and $M_{\Delta Z} = I - \Delta Z(\Delta Z'\Delta Z)^{-1}\Delta Z'$.Then, we have

$$T^{-2}\tilde{S}'M_{\Delta Z}\tilde{S} \to \sigma^2 \int_0^1 \underline{V}(r)^2 dr, \tag{2.14}$$

where $\underline{V}(r)$ is the projection of the process $V(r)$ on the orthogonal complement of the space spanned by the trigonometric function $dz = \left(1, dsin(2\pi kr), dcos(2\pi kr)\right)'$, with $r \in [0,1]$. Also, we have

$$\tilde{S}'M_{\Delta Z}\Delta\tilde{S} \to -\frac{1}{2}\sigma_\varepsilon^2. \tag{2.15}$$

This completes the proof. □

Remark 2.1. Theorem 2.1 shows that the LM-based cointegration test statistic has the same asymptotic distribution as the LM-based unit root test proposed by Enders and Lee (2012). This evidence supports that the distribution of developed test statistic does not depend on the regressors. Hence, regardless of the number of regressors, the critical values are the same as those in Table 1 by Enders and Lee (2012, p. 580). Nonetheless, we use a response surface function to compute the critical values of the univariate test statistics and the corresponding *p*-value for finite samples.

Remark 2.2. To control for the effects of autocorrelations, we use the parametric method since Westerlund and Edgerton (2007b) suggest that this method works best. Hence, we adopt our testing regression as follows

$$\Delta\tilde{S}_t = \phi\tilde{S}_{t-1} + constant + c_1\Delta\sin\left(\frac{2\pi kt}{T}\right) + c_2\Delta\cos\left(\frac{2\pi kt}{T}\right) + \sum_{j=1}^{p}\theta_j\Delta\tilde{S}_{t-j} + error \tag{2.16}$$

where p shows the number of lags. In the parametric approach, we need to determine the number of lags. Unless the optimal value of p is known, it can be found using the information criteria, such as the Akaike (AIC) or the Schwarz (BIC), or as suggested by Westerlund and Edgerton (2007b), p can be chosen to the largest integer less than $4(T/100)^{2/9}$.

Remark 2.3. Assumption 1(i) can be relaxed to allow for endogenous regressors. As suggested on Westerlund and Edgerton (2007b), instrumental variables that are uncorrelated

with the ε_t but highly correlated with Δx_t can be used instead of OLS to estimate the coefficients.

Remark 2.4. The number of the cumulative frequencies in (2.4) or the optimal value of the representative frequency in (2.5) can be determined by using the usual information criteria. Enders and Lee (2012) suggest using the cumulative frequency, $m = 1$ or 2, which can be suitable in many empirical applications. Enders and Lee (2012) have demonstrated that this can approximate multiple smooth breaks. Adding more cumulative frequencies can yield a more precise estimation of the underlying process. However, the use of a large number of cumulative frequencies is not recommended since it can lead to overfitting and a loss of power.

2.4. Response surface estimates

This section explains how we estimate the response surface function to obtain the critical values for finite sample and in case of the presence of serial correlation in the errors, and to find the *p*-values of the test statistic τ_{FLM} in (2.9). Since the test statistic has a non-standard limiting distribution, the corresponding *p*-values, denoted as $\tilde{p}$, must be approximated by simulation. As the model specification using the Fourier function is simpler, the response surface function using the frequency parameters is easy to estimate and it performs fairly well, even in finite samples. This feature can lead to good size and power properties since the uncertainty of estimating more parameters can be reduced.

We follow MacKinnon (1996, 2002) and estimate response surfaces to approximate $\tilde{p}$ for the τ_{FLM} statistic.[2] We conduct Monte Carlo simulations in order to obtain the empirical distribution of τ_{FLM} for a range of 26 sample sizes, *T*={30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 100, 125, 150, 175, 200, 225, 250, 300, 400, 450, 500, 2000} by using 100,000 replications. In that respect, we first specify the data generating process as in (2.1)–(2.3) and describe $y_t = d(t) + x_t'\beta + e_t, \quad x_t = x_{t-1} + w_t, \quad e_t = \rho e_{t-1} + \varepsilon_t$ where $w_t \sim N(0,1)$, $\varepsilon_t \sim N(0,1)$, $\beta \sim U[0,1]$, and $d(t)$ is described in (2.4) for cumulative frequencies and in (2.5)

[2] See Adda and Gonzalo (1996), MacKinnon (1996, 2002) and Hansen (1997) for the procedures to approximate *p*-values.

for a single frequency, where $U[\cdot]$ denotes the uniform distribution. We derive the empirical distribution of the τ_{FLM} statistic under the null hypothesis ($\rho = 1$) at 221 significance levels (l = 0.0001, 0.0002, 0.0005, 0.001, … , 0.01, 0.015, … , 0.985, 0.990, 0.991, 0.999, 0.9995, 0.9998, 0.9999). This implies that there will be 26 critical values of $\tilde{\tau}$ for each m at each l. In total, we estimate 17,238 response surfaces (26*3*221 combinations) for cumulative frequencies and 28,730 response surfaces (26*5*221 combinations) for a single frequency. The critical values are then fitted in the following functional form:[3]

$$CV_T^l = \theta_\infty^l + \sum_{i=1}^{4} \theta_i^l \left(\frac{1}{T}\right)^i + u^l \qquad (2.17)$$

where T shows the sample size and CV_T^l is the estimate of the critical value at the significance level l. Notice that as $T \to \infty$ the intercept θ_∞^l provides an estimate of the asymptotic critical values of the test statistic. Table 2.1 and Table 2.2 report the estimates of the response surface regression at the significance levels of 0.01, 0.05, and 0.10 for different combinations of cumulative Fourier frequencies and single Fourier frequency, respectively.

Table 2.1. Response surface estimates with no lags for the τ_{FLM} statistic with cumulative Fourier frequencies

m	$\theta \backslash l$	0.01	0.05	0.10
1	Intercept	-4.5327	-4.0285	-3.7611
	$1/T$	-18.7832	-6.7740	-7.3054
	$(1/T)^2$	484.9571	-222.7323	122.3586
	$(1/T)^3$	-22,437.24	7782.1482	-5025.4322
	$(1/T)^4$	299,108.9884	-95911.7649	64189.0691
1,2	Intercept	-5.2847	-4.7664	-4.5085
	$1/T$	-18.2107	-14.8202	-11.3036
	$(1/T)^2$	-456.3579	61.6918	103.1132
	$(1/T)^3$	16023.5562	-4661.2512	-7389.4905
	$(1/T)^4$	-318088.1961	38407.8744	102914.9564
1,2,3	Intercept	-5.8801	-5.3864	-5.1305
	$1/T$	-31.8987	-20.1330	-14.5215
	$(1/T)^2$	343.5199	146.8596	-109.1163
	$(1/T)^3$	-33195.9660	-20507.0569	-3633.8790
	$(1/T)^4$	483683.4869	348111.0380	74005.6566

Notes: m is the cumulative frequencies, l is the significance level, and T is the sample size.

[3] This functional form follows from Bai and Carrion-i-Silvestre (2009) and Otero and Smith (2012).

Table 2.2. Response surface estimates with no lags for the τ_{FLM} statistic with a single Fourier frequency

k	$\theta \backslash l$	0.01	0.05	0.10
1	Intercept	-4.5469	-4.0359	-3.7690
	$1/T$	-14.0281	-6.3844	-4.6923
	$(1/T)^2$	-123.3384	-154.3840	-126.0358
	$(1/T)^3$	6248.9961	4245.4362	5700.6106
	$(1/T)^4$	-144986.4646	-74291.3512	-114536.4060
2	Intercept	-4.1168	-3.5413	-3.2182
	$1/T$	-20.1208	-3.5002	-1.1662
	$(1/T)^2$	1333.5146	-2.7213	-1.7508
	$(1/T)^3$	-60919.4118	2120.4562	2341.9635
	$(1/T)^4$	915978.5438	-49275.2552	-56635.1610
3	Intercept	-3.9280	-3.3069	-2.9871
	$1/T$	-2.2363	1.8759	4.0829
	$(1/T)^2$	-439.8098	-272.8203	-263.2832
	$(1/T)^3$	19296.4565	13039.4161	12164.0152
	$(1/T)^4$	-262382.5596	-179541.0276	-170573.3998
4	Intercept	-3.8053	-3.1891	-2.8835
	$1/T$	-0.2703	-1.5750	0.6682
	$(1/T)^2$	-443.0816	452.2970	368.9701
	$(1/T)^3$	22446.5645	-21935.5913	-18612.2534
	$(1/T)^4$	-349387.1413	354564.5722	306902.0889
5	Intercept	-3.7422	-3.1418	-2.8407
	$1/T$	2.1020	4.4638	4.1285
	$(1/T)^2$	-466.6263	-290.3871	-85.5853
	$(1/T)^3$	20613.9322	14843.3905	4476.4196
	$(1/T)^4$	-266537.4505	-221242.6775	-53658.1444

Notes: *k* is the single frequency, *l* is the significance level, and *T* is the sample size.

Equation (2.17) does not consider the presence of serial correlation in the errors. As outlined above, we use the testing regression in (2.16). Note that even though the distribution of τ_{FLM} does not depend asymptotically on the lag order, p, it can affect the finite sample performance of τ_{FLM}. To account for the effect of autocorrelation, we chose the number of lags as $p = 4$ for $T \leq 50$, $p = 8$ for $50 < T \leq 80$, and $p = 12$ for $80 < T$. It is worthwhile to note here that the second-order polynomial equation proposed by Cheung and Lai (1995) appears to perform better. The response surface equation accounting for the lag order is given by[4]

[4] To evaluate how the critical values will change with different lag orders and sample sizes, we calculate the approximated critical values for each of the different values of T and p by using the response surface estimates. Tables 7.1 and 7.2 in Appendix A present the computed critical values at the 1, 5, and 10 percent levels.

$$CV_T^l = \theta_\infty^l + \sum_{i=1}^{2} \theta_i^l \left(\frac{1}{T}\right)^i + \sum_{i=1}^{2} \phi_i^l \left(\frac{p}{T}\right)^i + u^l. \quad (2.18)$$

Table 2.3 and Table 2.4 report the estimates of this response surface regression with lags at the significance levels of 0.01, 0.05, and 0.10 for the different combinations of the cumulative Fourier frequencies and single Fourier frequency, respectively.

Table 2.3. Response Surface Estimates with lags for the τ_{FLM} statistic with cumulative Fourier frequencies

m	$\theta \backslash l$	0.01	0.05	0.10
1	Intercept	-4.5424	-4.0315	-3.7648
	1/T	-13.7251	-8.8330	-5.7146
	$(1/T)^2$	-71.4008	-13.1514	-21.6457
	p/T	-0.0054	0.0770	-0.0095
	$(p/T)^2$	-0.5502	-0.1544	0.0365
1,2	Intercept	-5.2845	-4.7731	-4.5134
	1/T	-19.7628	-12.8554	-9.2293
	$(1/T)^2$	-200.9004	-102.2799	-83.6451
	p/T	0.0341	-0.0065	-0.0499
	$(p/T)^2$	-0.2674	0.0310	0.1053
1,2,3	Intercept	-5.8670	-5.3855	-5.1288
	1/T	-20.6930	-16.7169	-14.0379
	$(1/T)^2$	-547.6482	-250.6889	-166.2232
	p/T	-1.0054	-0.1682	-0.0514
	$(p/T)^2$	3.5255	0.5740	0.2795

Notes: See Table 1.

To calculate the *p*-value for a τ_{FLM} statistic, the interpolation procedure suggested by MacKinnon (1996, 2002) can be used:

$$\Phi^{-1}(l) = \beta_0 + \beta_1 \widehat{CV}^l + \beta_2 \left(\widehat{CV}^l\right)^2 + \beta_3 \left(\widehat{CV}^l\right)^3 + \nu^l \quad (2.19)$$

where Φ^{-1} is the inverse of the cumulative standard normal distribution of each of 221 significance levels, and $\widehat{CV}^l$ is the fitted critical value at quantile *l*. This model enables us to approximate the distribution of the test statistic in a small region around an estimated value of the τ_{FLM} statistic. We estimate (2.19) using the 11 points closest to a given value of the τ_{FLM} statistic. The approximate *p*-value, $\tilde{p}$, of the τ_{FLM} statistic then is given by

$$\tilde{p} = \Phi\left(\widehat{\beta_0} + \widehat{\beta_1}\tilde{\tau} + \widehat{\beta_2}\tilde{\tau}^2 + \widehat{\beta_3}\tilde{\tau}^3\right) \quad (2.20)$$

Table 2.4. Response Surface Estimates with lags for the τ_{FLM} statistic with a single Fourier frequency

k	$\theta \backslash l$	0.01	0.05	0.10
1	Intercept	-4.5535	-4.0280	-3.7717
	1/T	-18.0809	-8.2048	-6.8546
	$(1/T)^2$	39.7678	-26.5864	12.7550
	p/T	0.7905	-0.0101	0.2351
	$(p/T)^2$	-3.6647	-0.0528	-0.9547
2	Intercept	-4.1431	-3.5394	-3.2169
	1/T	-7.8390	-4.1972	-2.9038
	$(1/T)^2$	-58.8191	26.1989	69.3467
	p/T	0.0239	0.0769	0.1771
	$(p/T)^2$	-0.5481	-0.1018	-0.5153
3	Intercept	-3.9215	-3.3021	-2.9843
	1/T	-7.2379	-2.7524	1.2310
	$(1/T)^2$	68.5206	87.6564	39.2566
	p/T	0.2378	0.2259	0.1606
	$(p/T)^2$	-1.5418	-0.7093	-1.0983
4	Intercept	-3.8165	-3.1892	-2.8839
	1/T	-3.8552	0.5049	1.6432
	$(1/T)^2$	14.3624	53.7778	54.9300
	p/T	0.5363	0.0185	0.1080
	$(p/T)^2$	-2.3470	-0.1442	-0.3663
5	Intercept	-3.7111	-3.1318	-2.8349
	1/T	-5.9443	0.3475	1.9867
	$(1/T)^2$	139.4260	77.8972	59.8041
	p/T	-0.0464	0.0467	0.0007
	$(p/T)^2$	1.1354	0.0161	0.3202

Notes: See Table 2.

To illustrate how the response surface approach is used to approximate the p-value of an τ_{FLM}, let us consider the case where $m = 2$, $T = 200$, $p = 0$ and suppose that we have $\tau_{FLM} = -4.840$ (which is equal to the 5 percent critical value for the τ_{FLM} test). We first find the $\widehat{CV}^l$ and estimate β's in (2.20) by employing the $\widehat{CV}^l$. The approximated p-value is obtained as $p = \Phi\,(2.364 + (-0.413) * (-4.840) + (-0.416) * (-4.840)\text{^}2 + (-0.033) * (-4.840)\text{^}3)$, which is 0.050. Hence, this example shows that the response surface approach approximates the p-value properly.

2.5. Small sample simulations

This section examines the small sample properties of the proposed testing procedure through Monte Carlo simulations. In these simulations, our focus is on the performance of

the τ_{FLM} statistic in the presence of various types of breaks in deterministic term. The general form of DGP is given by

$$y_t = d(t) + x_t'\beta + e_t \qquad t = 1, \dots T \tag{2.21}$$

$$x_t = x_{t-1} + w_t \tag{2.22}$$

$$e_t = \rho e_{t-1} + \varepsilon_t \tag{2.23}$$

with β is an h-dimensional vector of ones, $\rho = 1$ for size analysis, and $\rho = 0.8$ for power analysis. Following Westerlund and Edgerton (2007b), we consider two cases for the Monte Carlo simulations. In the first, ε_t follows a first-order moving average (MA) process and it is defined as $\varepsilon_t = \upsilon_t + \theta\upsilon_{t-1}$ with $\theta = \{0.4, -0.4\}$. In the second case, ε_t follows a first-order autoregressive (AR) process, and it is described as $\varepsilon_t = \varphi\varepsilon_{t-1} + \upsilon_t$ with $\varphi = \{0.4, -0.4\}$. In all scenarios, we have $\upsilon_t \sim N(0,1)$, $w_t \sim N(0,1)$ and we use one regressor since the distribution of test statistic does not depend on the number of regressors.[5] Moreover, we compare the proposed test with no break and sharp break LM-based tests of Westerlund and Edgerton (2007b) for all cases. t_N and ϕ_N tests are based on the no break model and t_C and ϕ_C tests are based on the level break model (see Westerlund and Edgerton, 2007b). For the τ_{FLM} test with estimated frequency, we choose the maximum number of cumulative frequencies, m_{max}, is 3.[6] Monte Carlo simulations are carried out with 10,000 replications at the 5 percent level of significance.

To analyze the performance of the test statistics for various types of breaks, we consider five experiments. We define $d(t) = \{\}$ for no break experiment. In the sharp break experiment, we follow Westerlund and Edgerton (2007b) and describe $d(t) = \alpha_1 D_t$ where $\alpha_1 = 3$, $D_t = 1$ if $t > 0.5T$ and zero otherwise. Then, we proceed with the Fourier break experiment and indicate $d(t) = \alpha_1 \sin\left(\frac{2\pi kt}{T}\right) + \alpha_2 \cos\left(\frac{2\pi kt}{T}\right)$ where $\alpha_1 = \alpha_2 = 3$, $k = 1$. Moreover, we also consider logistic smooth transition autoregressive (LSTAR) and exponential smooth transition autoregressive (ESTAR) breaks, and we define $d(t) = 3/\left(1 + e^{0.05(t-\lambda T)}\right)$ for LSTAR break experiment and $d(t) = 2\left(1 - e^{(-0.0003)(t-\lambda T)^2}\right)$ for ESTAR break experiment, where $\lambda \sim U[0.1,1]$ and $U[\cdot]$ denotes the uniform distribution.

[5] To perform a robustness check, we consider different number of regressors, *h*, and obtain the size and power properties that are similar to those reported in Tables 2.5 and 2.6. The simulation results for $h = (2,3,4,5)$ are available in Tables 7.3-7.11 in the Appendix A.

[6] We estimate *m* sequentially by checking the joint significance of the Fourier coefficients at the 10% significance level of the F-test, starting from a maximum value of, say, $j = m_{max}$. If these coefficients are insignificant, we use fewer cumulative frequencies with $j = (m_{max} - 1)$ and check the significance of the corresponding coefficients.

Figure 2.1 presents the illustrations of sharp, LSTAR, ESTAR, and Fourier breaks in the DGPs.

Table 2.5 reports the size properties of the LM-based cointegration test statistics. Almost all the τ_{FLM} tests with fixed frequency or estimated frequency are very close to 5% nominal size, regardless of type of breaks and autocorrelation process. On the other hand, t_N and t_C tests perform well with only small size distortions for T=100 in all experiment except Fourier break. When we consider Fourier breaks in DGP, they are undersized especially for negative values of φ and θ. ϕ_N and ϕ_C are undersized when φ or θ are positive and oversized when φ or θ are negative.

Table 2.6 shows the power properties of the LM-based cointegration test statistics. As discussed by Becker et al. (2006), a high number of frequency terms decreases the degrees of freedom and can lead to an over-fitting problem. Hence, it leads to a loss of power. According to the results, this situation is valid for τ_{FLM} since the power of τ_{FLM} statistic decreases as *m* increases. However, τ_{FLM} statistics with $m = 1$ and estimated frequency have good power properties especially for smooth breaks. Additionally, the power of τ_{FLM} statistic rises as T increases, which is presumably reflecting the test′s consistency. While the test statistic proposed by Westerlund and Edgerton (2007b) is powerful especially for sharp breaks, they have power loss in the presence of Fourier breaks.

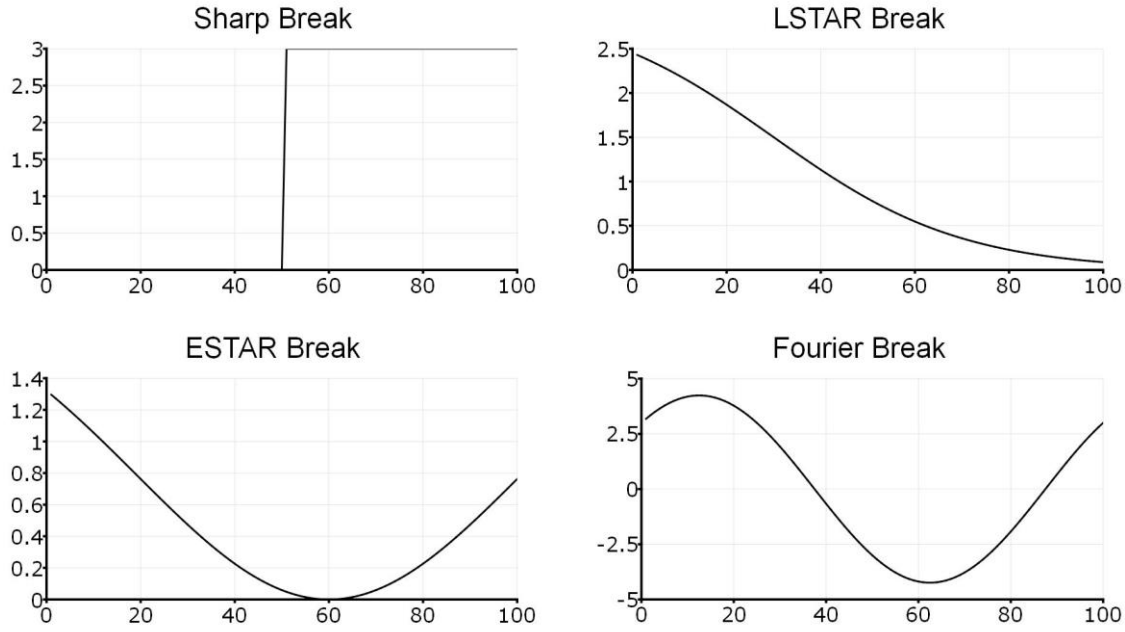


Figure 2. 1. Illustrations of Sharp, LSTAR, ESTAR, and Fourier breaks in the DGPs

Table 2.5. Size properties of the LM-based cointegration test statistics

B. Type	θ	φ	T	$\tau_{FLM}(1)$	$\tau_{FLM}(2)$	$\tau_{FLM}(3)$	$\tau_{FLM}(\hat{m})$	t_C	ϕ_C	t_N	ϕ_N
No break	0	0	100	0.042	0.050	0.056	0.048	0.065	0.078	0.063	0.078
			200	0.046	0.050	0.060	0.050	0.057	0.060	0.056	0.061
			300	0.046	0.048	0.055	0.049	0.046	0.060	0.047	0.059
	0.4	0	100	0.044	0.047	0.057	0.057	0.055	0.026	0.057	0.026
			200	0.052	0.046	0.049	0.062	0.058	0.020	0.058	0.019
			300	0.046	0.051	0.060	0.060	0.052	0.015	0.052	0.015
	-0.4	0	100	0.047	0.051	0.053	0.047	0.061	0.246	0.062	0.245
			200	0.057	0.054	0.066	0.057	0.053	0.232	0.051	0.237
			300	0.048	0.048	0.059	0.048	0.042	0.224	0.043	0.224
	0	0.4	100	0.046	0.053	0.066	0.067	0.058	0.014	0.057	0.013
			200	0.045	0.049	0.058	0.070	0.052	0.008	0.054	0.008
			300	0.048	0.051	0.055	0.064	0.050	0.005	0.048	0.005
	0	-0.4	100	0.044	0.047	0.054	0.044	0.063	0.180	0.061	0.182
			200	0.044	0.050	0.057	0.044	0.046	0.158	0.045	0.154
			300	0.047	0.049	0.051	0.048	0.055	0.161	0.054	0.163
Sharp	0	0	100	0.047	0.050	0.060	0.052	0.055	0.073	0.056	0.079
			200	0.048	0.048	0.052	0.053	0.053	0.061	0.053	0.061
			300	0.044	0.051	0.054	0.048	0.056	0.062	0.058	0.065
	0.4	0	100	0.043	0.050	0.060	0.056	0.061	0.022	0.059	0.022
			200	0.046	0.046	0.054	0.059	0.053	0.016	0.053	0.018
			300	0.050	0.054	0.059	0.067	0.048	0.014	0.045	0.012
	-0.4	0	100	0.042	0.042	0.043	0.042	0.060	0.236	0.053	0.204
			200	0.050	0.050	0.052	0.050	0.054	0.238	0.054	0.228
			300	0.051	0.050	0.050	0.051	0.050	0.224	0.050	0.217
	0	0.4	100	0.043	0.051	0.057	0.057	0.066	0.012	0.061	0.015
			200	0.049	0.054	0.062	0.068	0.048	0.005	0.047	0.007
			300	0.051	0.053	0.057	0.069	0.054	0.006	0.052	0.006
	0	-0.4	100	0.040	0.049	0.046	0.041	0.059	0.178	0.055	0.164
			200	0.046	0.048	0.053	0.046	0.050	0.162	0.056	0.160
			300	0.049	0.050	0.057	0.050	0.057	0.163	0.052	0.156
Fourier	0	0	100	0.043	0.045	0.054	0.047	0.030	0.016	0.031	0.017
			200	0.044	0.055	0.056	0.051	0.028	0.021	0.028	0.021
			300	0.042	0.055	0.057	0.046	0.035	0.033	0.035	0.034
	0.4	0	100	0.047	0.048	0.054	0.062	0.045	0.010	0.047	0.009
			200	0.051	0.050	0.057	0.062	0.045	0.010	0.044	0.010
			300	0.055	0.058	0.060	0.070	0.042	0.009	0.041	0.010
	-0.4	0	100	0.051	0.057	0.054	0.051	0.019	0.010	0.018	0.011
			200	0.047	0.051	0.057	0.047	0.017	0.043	0.017	0.045
			300	0.046	0.053	0.055	0.046	0.019	0.065	0.021	0.067
	0	0.4	100	0.042	0.056	0.064	0.061	0.041	0.006	0.041	0.006
			200	0.054	0.060	0.067	0.076	0.045	0.006	0.045	0.005
			300	0.053	0.053	0.061	0.075	0.043	0.004	0.043	0.004
	0	-0.4	100	0.048	0.046	0.054	0.048	0.021	0.017	0.024	0.017
			200	0.043	0.046	0.049	0.043	0.023	0.042	0.023	0.041

			300	0.047	0.054	0.058	0.047	0.029	0.063	0.028	0.063
LSTAR	0	0	100	0.049	0.047	0.050	0.050	0.055	0.133	0.054	0.135
			200	0.044	0.046	0.044	0.045	0.049	0.161	0.047	0.159
			300	0.050	0.050	0.053	0.051	0.049	0.179	0.050	0.180
	0.4	0	100	0.042	0.049	0.054	0.049	0.055	0.066	0.057	0.066
			200	0.052	0.052	0.054	0.055	0.055	0.084	0.054	0.082
			300	0.052	0.059	0.058	0.054	0.050	0.090	0.049	0.091
	-0.4	0	100	0.059	0.065	0.060	0.059	0.047	0.255	0.049	0.266
			200	0.083	0.096	0.096	0.083	0.047	0.341	0.053	0.353
			300	0.078	0.083	0.085	0.078	0.050	0.377	0.050	0.385
	0	0.4	100	0.044	0.054	0.066	0.055	0.058	0.043	0.060	0.041
			200	0.051	0.049	0.057	0.058	0.059	0.048	0.058	0.047
			300	0.050	0.053	0.058	0.055	0.049	0.053	0.050	0.053
	0	-0.4	100	0.042	0.040	0.051	0.042	0.055	0.209	0.050	0.213
			200	0.053	0.058	0.065	0.053	0.045	0.258	0.046	0.265
			300	0.059	0.059	0.059	0.059	0.048	0.300	0.049	0.305
ESTAR	0	0	100	0.046	0.050	0.054	0.047	0.058	0.119	0.060	0.121
			200	0.053	0.048	0.056	0.053	0.051	0.150	0.052	0.151
			300	0.051	0.045	0.047	0.052	0.052	0.149	0.049	0.148
	0.4	0	100	0.049	0.050	0.062	0.057	0.066	0.063	0.062	0.062
			200	0.049	0.049	0.057	0.053	0.054	0.074	0.055	0.074
			300	0.047	0.058	0.057	0.054	0.052	0.071	0.049	0.071
	-0.4	0	100	0.056	0.057	0.060	0.056	0.066	0.306	0.067	0.308
			200	0.075	0.079	0.090	0.075	0.064	0.377	0.065	0.379
			300	0.062	0.063	0.072	0.062	0.049	0.373	0.049	0.380
	0	0.4	100	0.047	0.053	0.064	0.058	0.065	0.032	0.065	0.032
			200	0.048	0.054	0.057	0.055	0.052	0.038	0.053	0.038
			300	0.049	0.058	0.059	0.060	0.051	0.040	0.050	0.040
	0	-0.4	100	0.044	0.046	0.052	0.044	0.062	0.232	0.058	0.240
			200	0.048	0.051	0.056	0.048	0.051	0.278	0.051	0.281
			300	0.046	0.049	0.055	0.046	0.049	0.279	0.049	0.286

Notes: θ refers to the MA parameter and φ refers to the AR parameter. $\tau_{FLM}(1)$, $\tau_{FLM}(2)$, $\tau_{FLM}(3)$ show τ_{FLM} statistic obtained by fixed frequencies with m=1, m=2, and m=3, respectively. τ_{FLM} statistic with estimated frequency is illustrated as $\tau_{FLM}(\hat{m})$. t_N and ϕ_N tests are based on no break model and t_C and ϕ_C tests are based on the level break model developed by Westerlund and Edgerton (2007b).

Table 2.6. Power properties of the LM-based cointegration test statistics

B. Type	θ	φ	T	$\tau_{FLM}(1)$	$\tau_{FLM}(2)$	$\tau_{FLM}(3)$	$\tau_{FLM}(\widehat{m})$	t_C	ϕ_C	t_N	ϕ_N
No break	0	0	100	0.149	0.095	0.085	0.149	0.247	0.458	0.317	0.491
			200	0.615	0.390	0.269	0.615	0.746	0.965	0.738	0.969
			300	0.890	0.721	0.552	0.890	0.901	0.903	0.946	0.951
	0.4	0	100	0.146	0.089	0.076	0.148	0.253	0.587	0.257	0.522
			200	0.579	0.346	0.233	0.579	0.744	0.498	0.808	0.491
			300	0.887	0.703	0.537	0.887	0.940	0.964	0.951	0.971
	-0.4	0	100	0.165	0.101	0.083	0.165	0.264	0.869	0.290	0.892
			200	0.652	0.454	0.323	0.652	0.643	0.826	0.730	0.879
			300	0.880	0.742	0.593	0.880	0.825	1.000	0.879	1.000
	0	0.4	100	0.135	0.091	0.080	0.140	0.317	0.055	0.299	0.057
			200	0.556	0.335	0.222	0.556	0.788	0.750	0.827	0.758
			300	0.875	0.664	0.482	0.875	0.962	0.996	0.967	0.997
	0	-0.4	100	0.149	0.088	0.076	0.149	0.274	0.364	0.297	0.418
			200	0.598	0.396	0.269	0.598	0.724	0.997	0.766	0.999
			300	0.868	0.712	0.538	0.868	0.854	0.961	0.891	0.979
Sharp	0	0	100	0.119	0.085	0.072	0.119	0.254	0.466	0.221	0.345
			200	0.488	0.328	0.236	0.488	0.736	0.968	0.526	0.905
			300	0.793	0.635	0.479	0.793	0.891	0.887	0.828	0.830
	0.4	0	100	0.137	0.093	0.084	0.141	0.260	0.595	0.211	0.449
			200	0.532	0.334	0.232	0.532	0.736	0.513	0.676	0.360
			300	0.840	0.660	0.496	0.840	0.941	0.966	0.889	0.929
	-0.4	0	100	0.095	0.074	0.064	0.095	0.263	0.878	0.125	0.709
			200	0.411	0.303	0.231	0.411	0.654	0.831	0.393	0.617
			300	0.690	0.572	0.466	0.690	0.812	0.998	0.584	0.995
	0	0.4	100	0.126	0.088	0.078	0.131	0.306	0.057	0.244	0.047
			200	0.509	0.307	0.213	0.511	0.792	0.741	0.731	0.635
			300	0.824	0.622	0.449	0.824	0.960	0.995	0.927	0.987
	0	-0.4	100	0.108	0.082	0.075	0.108	0.259	0.357	0.157	0.202
			200	0.424	0.298	0.216	0.424	0.719	0.996	0.486	0.988
			300	0.712	0.572	0.448	0.712	0.859	0.963	0.675	0.901
Fourier	0	0	100	0.151	0.096	0.085	0.152	0.006	0.000	0.010	0.001
			200	0.630	0.403	0.264	0.630	0.002	0.005	0.001	0.004
			300	0.898	0.727	0.557	0.898	0.004	0.000	0.011	0.000
	0.4	0	100	0.150	0.102	0.083	0.153	0.016	0.014	0.016	0.008
			200	0.584	0.360	0.234	0.584	0.022	0.000	0.033	0.000
			300	0.892	0.705	0.515	0.892	0.063	0.022	0.070	0.024
	-0.4	0	100	0.170	0.104	0.085	0.170	0.006	0.002	0.005	0.002
			200	0.654	0.454	0.322	0.654	0.001	0.000	0.001	0.000
			300	0.879	0.763	0.608	0.879	0.001	0.130	0.002	0.115
	0	0.4	100	0.138	0.100	0.088	0.146	0.035	0.001	0.031	0.000
			200	0.570	0.334	0.227	0.571	0.082	0.014	0.101	0.014
			300	0.865	0.645	0.464	0.865	0.201	0.370	0.196	0.352
	0	-0.4	100	0.147	0.097	0.074	0.147	0.008	0.000	0.008	0.000
			200	0.603	0.389	0.264	0.603	0.003	0.102	0.003	0.093

			300	0.875	0.715	0.544	0.875	0.001	0.000	0.001	0.000
LSTAR	0	0	100	0.153	0.092	0.081	0.153	0.151	0.401	0.203	0.444
			200	0.662	0.424	0.292	0.662	0.467	0.907	0.477	0.929
			300	0.922	0.770	0.599	0.922	0.631	0.797	0.750	0.896
	0.4	0	100	0.147	0.097	0.086	0.147	0.178	0.650	0.184	0.605
			200	0.627	0.394	0.253	0.627	0.568	0.658	0.644	0.669
			300	0.916	0.745	0.555	0.916	0.809	0.966	0.849	0.981
	-0.4	0	100	0.182	0.117	0.096	0.182	0.119	0.670	0.134	0.674
			200	0.762	0.586	0.442	0.762	0.328	0.587	0.388	0.649
			300	0.928	0.836	0.704	0.928	0.450	0.985	0.516	0.992
	0	0.4	100	0.152	0.094	0.077	0.153	0.242	0.122	0.234	0.130
			200	0.612	0.376	0.255	0.612	0.670	0.851	0.724	0.870
			300	0.888	0.692	0.516	0.888	0.869	0.993	0.885	0.997
	0	-0.4	100	0.164	0.102	0.076	0.164	0.140	0.249	0.145	0.292
			200	0.675	0.448	0.313	0.675	0.397	0.975	0.444	0.981
			300	0.907	0.791	0.631	0.907	0.530	0.843	0.586	0.907
ESTAR	0	0	100	0.146	0.101	0.078	0.146	0.235	0.535	0.315	0.577
			200	0.625	0.408	0.284	0.625	0.689	0.978	0.702	0.983
			300	0.900	0.742	0.567	0.900	0.837	0.919	0.909	0.963
	0.4	0	100	0.148	0.094	0.080	0.149	0.238	0.712	0.245	0.669
			200	0.625	0.392	0.258	0.625	0.718	0.772	0.787	0.785
			300	0.908	0.722	0.535	0.908	0.923	0.988	0.942	0.991
	-0.4	0	100	0.170	0.114	0.089	0.170	0.247	0.863	0.262	0.881
			200	0.705	0.522	0.371	0.705	0.586	0.820	0.678	0.869
			300	0.899	0.812	0.672	0.899	0.721	0.997	0.797	0.999
	0	0.4	100	0.140	0.099	0.082	0.143	0.328	0.135	0.315	0.141
			200	0.593	0.365	0.242	0.593	0.776	0.913	0.823	0.928
			300	0.879	0.676	0.498	0.879	0.940	0.999	0.947	0.999
	0	-0.4	100	0.143	0.091	0.081	0.143	0.259	0.394	0.282	0.454
			200	0.636	0.430	0.305	0.636	0.659	0.995	0.700	0.997
			300	0.889	0.751	0.591	0.889	0.774	0.954	0.825	0.971

Notes: See Table 2.5.

2.6. Concluding remarks

In this study, we propose a new cointegration test in the presence of unknown forms of structural changes. The test extends Enders and Lee (2012) unit root test to multivariate cointegration context by applying the LM principle, and it can admit serially correlated and heteroskedastic errors, deterministic trends, and smooth structural breaks. Furthermore, the test uses the Fourier approximation suggested by Gallant (1981) to accommodate structural breaks. The advantage of this approach is that a small number of Fourier terms mimics well for various types of breaks and it does not require a priori information such as the number, location, or form of breaks.

The asymptotic distribution of the proposed test statistic is free of nuisance parameters other than the Fourier frequency. Moreover, the asymptotic analysis reveals that the new test is independent of the regressors. Therefore, we do not need to compute the critical values since the distribution of test statistic is identical to that obtained by Enders and Lee (2012). Nonetheless, making the testing procedure relatively simple and easy to implement, we present the convenient response surface estimates to obtain finite-sample critical values and corresponding p-values.

Monte Carlo simulations are conducted to evaluate the finite-sample size and power of the suggested test. Our simulation results reveal that the proposed test performs reasonably well for different specifications of structural breaks, with improved size and power properties.

3. TESTING FOR COINTEGRATION IN DEPENDENT PANELS WITH SMOOTH STRUCTURAL BREAKS

3.1. Introduction

Panel data analysis has become popular in recent decades as it offers many advantages that are absent under the framework of pure cross-sectional or time series analysis. While the usual univariate cointegration tests have low power, the inherently larger sample size of panel data naturally improves the power of panel-based tests. As such, Kao (1999), Pedroni (1999, 2004), and Larsson et al. (2001), among others, have popularized the so-called first-generation panel cointegration tests that led to numerous empirical applications and subsequent extensions of these tests.

It is well known that the first-generation panel cointegration tests are based on the crucial assumption that the errors are cross-sectionally independent. This is unrealistic since the international business cycle, the proliferation of technology that connects once-isolated economies, and many other stylized economic facts show evidence of cross-sectional dependence. Subsequently, the second-generation panel cointegration tests allow for cross-sectional dependence. Gengenbach et al. (2006) and Westerlund (2008) use the PANIC (panel analysis of non-stationarity in idiosyncratic and common components) approach of Bai and Ng (2004, 2010), while Gengenbach et al. (2016) and Banerjee and Carrion-i-Silvestre (2017) employ the cross-section average (CA) augmentation approach of Pesaran (2006) to control cross-correlations. Additionally, several first-generation panel cointegration studies relax the assumption of cross-sectional independence using the bootstrap approach in their testing procedures (see Westerlund 2007; Westerlund and Edgerton, 2007a; Hanck, 2009). The PANIC approach does not affect the asymptotic behavior of test statistics, while the CA approach does. Another advantage of the PANIC approach is that it is not necessary to assume that the factors are stationary or non-stationary. In practice, this assumption is rather restrictive, but the PANIC approach overcomes this problem by decomposing the error term. On the other hand, the bootstrap approach considers the cross-section dependency of the general form and avoids the possibility of misspecification (Hadri et al., 2015). However, it does not provide information on the dependency structure. Furthermore, during the last decades, the importance of using factor

models to take into account cross-correlations has increased (Bai and Ng, 2004). The idea behind factor models is that global shocks or common factors might influence each cross-section differently. It is reasonable to predict that some common factors, depending on their inherent characteristics, will affect cross-sections in the panel with varying intensities (Karabiyik et al., 2019).

One important additional issue is associated with structural breaks in light of the contribution of Perron (1989), who showed that the usual unit root tests suffer a loss of power when ignoring breaks. Hao (1996) shows that neglecting structural breaks lead to size distortions and deceptive inference when testing for cointegration. As a result, the panel cointegration literature focuses on accounting for structural breaks. However, the cointegration tests with breaks for dependent panels have been relatively limited. Two of the important studies in this field, Westerlund and Edgerton (2008) and Banerjee and Carrion-i-Silvestre (2015), suggest panel cointegration tests in the presence of multiple structural changes and common dynamic factors. In particular, they use dummy variables to control for the effects of structural changes and apply the PANIC procedure to deal with cross-sectional dependence. Another important contribution in this area is Olayeni et al. (2021), who propose a panel cointegration test with smooth breaks. They use the Fourier approximation to capture breaks and utilize the bootstrap approach to consider the cross-correlations in the panel.

There are several operational advantages of using the Fourier approach instead of the dummy variable method. First the model specification using Fourier terms can be quite parsimonious, and, as such, it yields a test with good size and power properties. Cointegration tests that use dummy variables to capture multiple structural breaks inherently require estimation of more parameters, resulting in a significant loss of power. By comparison, the Fourier approach has fewer parameters to estimate, even when multiple structural breaks are present, and this results in a more powerful test. Second, the Fourier terms can fit quite well for various types of breaks observed in real-world economic data, and it can approximate many types of smooth breaks. For instance, it works very well for smooth breaks including logistic smooth transition autoregressive and exponential smooth transition autoregressive breaks. This approach might not be the most effective one, though, when there are sharp trend breaks or abrupt level breaks. The tests utilizing dummy variables

could be more suited in such situation. So, our study can complement that of Westerlund and Edgerton (2008).

This study proposes new Lagrange Multiplier (LM)-based cointegration tests with structural breaks for dependent panels. Our tests differ from Westerlund and Edgerton (2008) and Olayeni et al. (2021) in considering structural breaks and dealing with cross-sectional dependence. We apply the Fourier approach to capture multiple structural breaks. Using the dummy variable approach needs to employ multiple dummies to consider multiple structural changes. Thus, the power of the test reduces considerably, as the number of dummy variables increases. Alternatively, the Fourier approach can capture unknown multiple structural changes by a using small number of frequencies (Nazlioglu et al., 2022). Moreover, we utilize the PANIC approach of Bai and Ng (2004) and show that our panel cointegration tests do not depend on the factor structure or unknown forms of cross-correlations of the error terms. The asymptotic results of our tests also indicate that the distribution of test statistics only depends on the Fourier frequency and is independent of the number of regressors. Finally, we perform Monte Carlo simulations to evaluate the finite-sample size and power of the suggested tests. The simulation results reveal that the proposed tests have good size and power properties for stationary or non-stationary common factors under the different types of structural breaks.

Since the test statistics are derived from the LM-based unit root tests suggested by Schmidt and Phillips (1992) and Enders and Lee (2012), they can accommodate (i) heteroskedastic and serially correlated errors, (ii) unknown multiple structural breaks, and (iii) cross-sectional dependence. Finally, we provide a response surface function to compute the critical values of the univariate LM cointegration test statistics and the corresponding p-value for finite samples, which facilitates the use of testing procedure in empirical studies.

The rest of the paper is as follows. The next section describes the model and testing procedure. Section 3 illustrates the asymptotic distribution of the proposed test statistics. Section 4 is devoted to small sample analysis. The concluding remarks are finally presented in section 5.

3.2. Model and testing procedure

The data generating process (DGP) can be written as follows:

$$y_{it} = d_i(t) + x'_{it}\beta_i + v_{it} \tag{3.1}$$

$$v_{it} = \pi'_i F_t + e_{it} \tag{3.2}$$

$$(I - L)F_t = C(L)u_t \tag{3.3}$$

$$x_{it} = x_{it-1} + \Gamma_i(L)w_{it} \tag{3.4}$$

$$(1 - \rho_i L)e_{it} = H_i(L)\varepsilon_{it} \tag{3.5}$$

with $t = 1, \dots T$ and $i = 1, \dots N$, where $C(L) = \sum_{j=0}^{\infty} C_j L^j$, $\Gamma_i(L) = \sum_{j=0}^{\infty} \Gamma_{ij} L^j$ and $H_i(L) = \sum_{j=0}^{\infty} \mathrm{H}_{ij} L^j$, and L is the lag operator. Here, y_{it} is the dependent variable, x_{it} is an h-dimensional vector of regressors, $d_i(t)$ represents the deterministic terms, and F_t is an r-dimensional vector of unobserved common factors, with π_i the vector of factor loadings.

As shown in Bai and Ng (2004), the factor F_t can be $I(0)$ or $I(1)$. If $C(1) = 0$, then F_t is $I(0)$. If $C(1)$ has full rank, then F_t is $I(1)$. If $C(1) \neq 0$, but not full rank, then some components of F_t are $I(0)$ and some are $I(1)$. To estimate the factors, we adopt the PANIC approach of Bai and Ng (2004). The benchmark procedure of our test in estimating the factors is similar. We also adopt assumptions like those suggested in the aforementioned papers. Let $A < \infty$ be a generic positive number independent of T and N. Further, $\|U\| = trace(U'U)^{1/2}$. Then

Assumption I:

(i) $E(\varepsilon_{it}) = 0$

(ii) $\sup_t E|\varepsilon_{it}|^{\delta+\omega} < \infty$ for $\delta > 0, \omega > 0$

(iii)$\sigma_i^2 = lim \frac{1}{T} E(S_{iT}^2), \sigma_i^2 > 0$ where $S_{iT} = \sum_{t=1}^{T} \varepsilon_{it}$.

(iv)$\{\varepsilon_{it}\}_1^{\infty}$ is strong mixing with mixing parameters α_m satisfying $\sum_{-\infty}^{\infty} {\alpha_m}^{1-\delta} < 0$.

Assumption II:

(i) $||\pi_i|| \leq A$ for non-random π_i.

(ii) $E||\pi_i||^4 \leq A$ for random π_i.

(iii) $\frac{1}{N}\sum_{i=1}^{N} \pi_i {\pi_i}' \rightarrow \Sigma_\pi$.

Assumption III:

(i) $u_t \sim iid(0, \Sigma_u)$, $E||u_t||^4 \leq A$.

(ii) $var(\Delta F_t) = \sum_{j=0}^{\infty} C_j \Sigma_u C_j' > 0$.

(iii) $\sum_{j=0}^{\infty} j||C_j|| < A$, $C(1)$ has rank r_1, $0 \leq r_1 \leq r$.

Assumption IV:

(i) $\varepsilon_{it} \sim iid(0, \Sigma_{\varepsilon_i})$, for each i, $E|\varepsilon_{it}|^8 \leq A$, $\sum_{j=0}^{\infty} j|H_{ij}| < A$ with $H_i(1)^2 \sigma_i^2 > 0$.

(ii) ε_{it} are independent over i.

Assumption V:

The errors $\varepsilon_{it}, u_t, w_{it},$ and the loadings π_i are mutually independent groups.

Assumption VI:

$E\|F_0\| \leq A$, $E|e_{i0}| \leq A$, and $E\|x_{i0}\| \leq A$ for every $i = 1, \ldots, N$.

Assumption VII:

(i) $w_{it} \sim iid(0, \Sigma_{w_i})$, $E||w_{it}||^4 \leq A$.

(ii) The partial sum process of w_{it} satisfies an invariance principle as $T \to \infty \; \forall i$.

(iii) $var(\Delta x_{it}) = \sum_{j=0}^{\infty} \Gamma_j \Sigma_{w_i} \Gamma_j' > 0$.

(iv) $\sum_{j=0}^{\infty} j||\Gamma_j|| < A$, $\Gamma(1)$ has full rank.

Assumption VIII

$E(e_{it}|w_{it}) = 0$.

Assumption I embodies the general assumptions for residual-based cointegration tests. Assumption II is related to the conditions under which the factor loadings are identifiable. Assumption III specifies the factors' conditions. Weak autocorrelations and cross-sectional error term independence are permitted by Assumptions IV and V. Assumption VI is associated with the initial conditions. Conditions for the first differences of stochastic regressors are defined by Assumption VII. Finally, Assumption VIII is related to the exogeneity of the stochastic regressors. This assumption can be relaxed. As suggested in Banerjee and Carrion-i-Silvestre (2015), the dynamic ordinary least squares (DOLS) estimation method can be employed to account for endogeneity since v_{it} is orthogonal to w_{it} when re-defined v_{it} as Remark 3.4 in Westerlund and Edgerton (2008, p. 676-677).

To consider the possible breaks in the deterministic term $d_i(t)$. Therefore, we utilize the Fourier approximation, which is based on a variant of the Flexible Fourier Form by Gallant (1981) for capturing structural breaks. This approximation does not require prior knowledge of the number, dates, and form of breaks and captures structural breaks as a gradual/smooth process by using a small number of low-frequency components. Following

Enders and Lee (2012) and Nazlioglu et al. (2022), the Fourier function with an unknown form of breaks can be described as follows:

$$d_i(t) = \alpha_{0i} + \alpha_{1i}t + \sum_{k=1}^{m} \gamma_{1ik} sin\left(\frac{2\pi kt}{T}\right) + \sum_{k=1}^{m} \gamma_{2ik} cos\left(\frac{2\pi kt}{T}\right) \quad (3.6)$$

where *m* is the number of frequencies, γ_{1ik} and γ_{2ik} respectively measure the amplitude and displacement. As shown in Becker et al. (2006), a single Fourier frequency can mimic a variety of breaks in deterministic components regardless of date, number, and form of breaks. Hence, it can be defined as

$$d_i(t) \cong \alpha_{0i} + \alpha_{1i}t + \gamma_{1i}\sin\left(\frac{2\pi kt}{T}\right) + \gamma_{2i}\cos\left(\frac{2\pi kt}{T}\right) \quad (3.7)$$

where *k* denotes the frequency for the approximation. It is clear from equation (3.7) that if $\gamma_{1i} = \gamma_{2i} = 0$ the model reduces to the linear specification and the cointegration procedure with no structural breaks proposed by Westerlund and Edgerton (2008) is appropriate.

The purpose of this study is to test the null hypothesis of no cointegration against the alternative hypothesis of cointegration, which is equivalent to testing $H_0: \rho_i = 1, i = 1, \dots, N$ against $H_1: \rho_i < 1$, for some i's. We use the Lagrange Multiplier methodology of Schmidt and Philips (1992) and Enders and Lee (2012). By imposing the null restriction, we estimate the following ordinary least squares (OLS) regression using the first differences for each i-th cross-sectional unit:

$$\Delta y_{it} = \delta_{0i} + \sum_{k=1}^{m} \delta_{1ik} \Delta sin\left(\frac{2\pi kt}{T}\right) + \sum_{k=1}^{m} \delta_{2ik} \Delta cos\left(\frac{2\pi kt}{T}\right) + \Delta x_{it}'\beta_i + \Delta v_{it}. \quad (3.8)$$

Since we consider the dependent panels, we need to estimate the factors and their loadings. We utilize the method of principal components, and begin by taking first differences of equation (3.2),

$$\Delta v_{it} = \pi_i' \Delta F_t + \Delta e_{it}. \quad (3.9)$$

The estimated factors, $\Delta\hat{F}_t$, are the *r* eigenvectors corresponding to the *r* largest eigenvalues of the matrix $\Delta v \Delta v'$, with $\Delta v = [\Delta v_1, .., \Delta v_N]$, and the factor loadings are given

by $\widehat{\Pi} = \Delta v' \Delta \widehat{F}$ with $\widehat{\Pi} = [\hat{\pi}_1, \dots, \hat{\pi}_N]$.[7] Therefore, the residual can be computed using the estimated parameters and the factor terms, as in:

$$\Delta \hat{e}_{it} = \Delta y_{it} - \hat{\delta}_{0i} - \sum_{k=1}^{m} \hat{\delta}_{1ik} \Delta sin\left(\frac{2\pi kt}{T}\right) - \sum_{k=1}^{m} \hat{\delta}_{2ik} \Delta cos\left(\frac{2\pi kt}{T}\right) - \Delta x'_{it} \hat{\beta}_i - \hat{\pi}'_i \Delta \hat{F}_t. \quad (3.10)$$

Then, following Banerjee and Carrion-i-Silvestre (2015) and Nazlioglu et al. (2022), the detrended variable $\tilde{S}_{it}$ can be defined as $\tilde{S}_{it} = \sum_{s=2}^{t} \Delta \hat{e}_{is}$, where $\Delta \hat{e}_{is}$ is the residual from equation (3.10). Note that $\tilde{S}_{it} = 0$ when $t = 1$.

Finally, the testing regression[8] with detrended series can be written as follows for each cross-sectional unit:

$$\Delta \tilde{S}_{it} = \phi_i \tilde{S}_{it-1} + constant + \sum_{k=1}^{m} c_{1ik} \Delta sin\left(\frac{2\pi kt}{T}\right) + \sum_{k=1}^{m} c_{2ik} \Delta cos\left(\frac{2\pi kt}{T}\right) + error. \quad (3.11)$$

The null hypothesis of no cointegration versus the alternative of cointegration can be formulated equivalently as $H_0: \phi_i = 0$ for all $i's$ versus $H_1: \phi_i < 0$ for some i's. Hence, the LM test statistic is:

$$\tilde{\tau}_{FLM_i} = \frac{\tilde{\phi}_i}{SE(\tilde{\phi}_i)} \quad (3.12)$$

where $\tilde{\phi}_i$ and $SE(\tilde{\phi}_i)$ are the OLS estimate of ϕ_i and the estimated standard error of $\tilde{\phi}_i$ in equation (11), respectively.

3.3. Asymptotic distribution of test statistic

In this section, we first derive the limiting distribution of the univariate test statistic under the null hypothesis ($H_0: \phi_i = 0$). For simplicity, we consider a simplified version of the first step regression using a single representative frequency parameter, *k*, for all cross-sectional units. However, we point out that using heterogeneous frequencies with various m_i or k_i values does not change the properties of the test.

[7] Here, the number of factors can be determined by using the information criteria values of Bai and Ng (2002). See Remark 3.4 for details.

[8] Unlike the study of Westerlund and Edgerton (2008), this regression includes Fourier terms since tests statistics are affected by Fourier terms, as stated in the theorem shown in the next section.

Theorem 3.1: *Suppose that y_t and x_t are generated by the DGP in equations (3.1) – (3.5) with $\rho = 1$, and one adopts the first step testing regression equation (3.8). Under H_0 and Assumption I to VIII, as $T \to \infty$, then,*

$$\tilde{\tau}_{FLM} \to -\frac{1}{2}\left(\frac{\sigma_\varepsilon}{\sigma}\right)\left[\int_0^1 \underline{V}(r)^2 dr\right]^{-\frac{1}{2}} \tag{3.13}$$

where $\underline{V}(r)$ is the projection of the process $V(r)$ on the orthogonal complement of the space spanned by the trigonometric function $dz = \left(1, dsin(2\pi kr), dcos(2\pi kr)\right)'$, with $r \in [0,1]$.

Proof[9]*:* First, we are interested in the asymptotic distribution of the normalized partial sum process, $\frac{1}{\sqrt{T}}\tilde{S}_{i,t} = \frac{1}{\sqrt{T}}\sum_{s=2}^{t}\Delta\hat{e}_{is}$, where $\Delta\hat{e}_{is}$ is the residual from regression (3.8) – (3.10). The first step regression is given by

$$\Delta y_{it} = \delta_{0i} + \delta_{1i}\Delta\sin(2\pi kt/T) + \delta_{2i}\Delta\cos(2\pi kt/T) + \Delta x_{it}'\beta_i + \pi_i' f_t + \Delta e_{it} \tag{3.14}$$

with $f_t = \Delta F_t$. This expression can also be written as

$$\Delta y_i = X_i\delta_i + f\pi_i + \Delta e_i \tag{3.15}$$

where $\Delta y_i = (\Delta y_{i2}, \dots, \Delta y_{iT})'$, $X_i = (X_{i2}, \dots, X_{iT})'$ with $X_{it} = \left[1, \Delta\sin\left(\frac{2\pi kt}{T}\right), \Delta\cos\left(\frac{2\pi kt}{T}\right), \Delta x_{it}\right]'$, $\delta_i = (\delta_{0i}, \delta_{1i}, \delta_{2i}, \beta_i)'$, $f = (f_2, \dots, f_T)'$, $\pi_i = (\pi_{i1}, \dots, \pi_{ir})'$, and $\Delta e_i = (\Delta e_{i2}, \dots, \Delta e_{iT})'$.

Thus, the residual from (3.14) is

$$\begin{aligned}\Delta\hat{e}_{it} &= \Delta y_{it} - \Delta\hat{y}_{it} \\ &= \Delta y_{it} - \hat{X}_{it}'\hat{\delta}_i + f_t'\pi_i \\ &= \Delta e_{it} - X_{it}'\left(\hat{\delta}_i - \delta_i\right) + \left(X_{it}' - \hat{X}_{it}'\right)\hat{\delta}_i + f_t'\pi_i - \hat{f}_t'\hat{\pi}_i\end{aligned} \tag{3.16}$$

Then, the normalized partial sum process can be written as

$$\begin{aligned}\frac{1}{\sqrt{T}}\sum_{s=2}^{t}\Delta\hat{e}_{is} = \ & \frac{1}{\sqrt{T}}\sum_{s=2}^{t}\Delta e_{it} + \frac{1}{\sqrt{T}}\sum_{s=2}^{t}\left(f_s'\pi_i - \hat{f}_s'\hat{\pi}_i\right) \\ & - \frac{1}{\sqrt{T}}\sum_{s=2}^{t}X_{is}'\left(\hat{\delta}_i - \delta_i\right) + \frac{1}{\sqrt{T}}\sum_{s=2}^{t}\left(X_{is}' - \hat{X}_{is}'\right)\hat{\delta}_i\end{aligned} \tag{3.17}$$

We now evaluate each of the terms in (3.17).

[9] The proof here closely follows that of Bai and Ng (2004) and Nazlioglu et al (2022), demonstrating that the asymptotic distribution of the test with smooth breaks remains same even after controlling for factor structure using the PANIC procedure and is unaffected by the number of regressors.

(a) The first term follows:

$$\frac{1}{\sqrt{T}}\sum_{s=2}^{t}\Delta e_{it} \;\rightarrow\; W(r) \tag{3.18}$$

(b) For the second term, we use the results of Bai and Ng (2004)

$$f_s'\pi_i - \hat{f}_s'\hat{\pi}_i = -\epsilon_s H^{-1}\pi_i - f_s' a_i$$

with $a_i = \hat{\pi}_i - H^{-1}\pi_i$, and $\epsilon_s = \hat{f}_s - H' f_s$, for which we have

$$T^{-1/2}\|\textstyle\sum_{s=2}^{t}\epsilon_s H^{-1}\pi_i\| = o_p(1) \text{ and } T^{-1/2}\|\textstyle\sum_{s=2}^{t} f_s' a_i\| = o_p(1). \tag{3.19}$$

Bai and Ng (2004, p. 1141) note that the matrix H is defined as the projection of $\hat{f}_s$ onto the space spanned by the factors, f_s, where

$$Min(N,T)\frac{1}{T}\sum ||\hat{f}_s - Hf_s||^2 \;=\; O_p(1).$$

Therefore, the second term is asymptotically negligible, which is the key feature of the PANIC procedure.

$$\frac{1}{\sqrt{T}}\sum_{s=2}^{t}\left(f_s'\pi_i - \hat{f}_s'\hat{\pi}_i\right) = o_p(1) \tag{3.20}$$

(c) We can write the third term in (3.17) as

$$\frac{1}{\sqrt{T}}\sum_{s=2}^{t} X_{is}'\left(\hat{\delta}_i - \delta_i\right) = \left[\frac{1}{T}\sum_{s=2}^{t} X_{is}'\right]\sqrt{T}\left(\hat{\delta}_i - \delta_i\right) \tag{3.21}$$

We first note that $\hat{\delta}_i$ can be given as the coefficient of $\hat{X}_i$ from the regression of $(\Delta y_i - \hat{f}\hat{\pi}_i)$ on $\hat{X}_i$, where $\hat{\delta}_i = (\hat{X}_i'\hat{X}_i)^{-1}\hat{X}_i'(\Delta y_i - \hat{f}\hat{\pi}_i)$. We can express this as

$$\begin{aligned}\hat{\delta}_i &= (X_i'X_i)^{-1}X_i'(\Delta y_i - \hat{f}\hat{\pi}_i) + O_p(T^{-1})\\ &= (X_i'X_i)^{-1}X_i'(\Delta y_i - f\hat{\pi}_i) + (X_i'X_i)^{-1}X_i'(f\pi_i - \hat{f}\hat{\pi}_i) + O_p(T^{-1})\\ &= \delta_i + (X_i'X_i)^{-1}X_i'\Delta e_i + (X_i'X_i)^{-1}X_i'(f\pi_i - \hat{f}\hat{\pi}_i) + O_p(T^{-1}),\end{aligned}$$

for which we have used the results: $(\hat{X}_i'\hat{X}_i)^{-1}\hat{X}_i'\Delta y_i = (X_i'X_i)^{-1}X_i'\Delta y_i + O_p(T^{-1})$, $(\hat{X}_i'\hat{X}_i)^{-1}\hat{X}_i'\hat{f}\hat{\pi}_i = (X_i'X_i)^{-1}X_i'\hat{f}\hat{\pi}_i + O_p(T^{-1})$, and $\Delta y_i - f\pi_i = X_i\delta_i + \Delta e_i$. Thus, we have

$$\begin{aligned}\sqrt{T}\left(\hat{\delta}_i - \delta_i\right) = &\left(\tfrac{1}{T}X_i'X_i\right)^{-1}\tfrac{1}{\sqrt{T}}X_i'\Delta e_i\\ &+ \left(\tfrac{1}{T}X_i'X_i\right)^{-1}\tfrac{1}{\sqrt{T}}X_i'\left(f\pi_i - \hat{f}\hat{\pi}_i\right) + O_p\left(T^{-\frac{1}{2}}\right)\end{aligned} \tag{3.22}$$

(i) Using the first term on the right-hand side of (3.22), we have

$$\left[\frac{1}{T}\sum_{s=2}^{t} X_s{}'\right]\left(\frac{1}{T}X_i{}'X_i\right)^{-1} X_i'\Delta e_i \to \sigma\begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & Q_{14} \\ Q_{12}' & Q_{22} & Q_{23} & Q_{24} \\ Q_{13}' & Q_{23}' & Q_{33} & Q_{34} \\ Q_{14}' & Q_{24}' & Q_{34}' & Q_{44} \end{bmatrix} \tag{3.23}$$

where

$Q_{11} = W(1)$,

$Q_{12} = (2\pi k)\left[W(1) + (2\pi k)\int_0^1 sin(2\pi kr)W(r)dr\right]$,

$Q_{13} = (2\pi k)^2\left[\int_0^1 cos(2\pi kr)W(r)dr\right]$,

$Q_{14} = 0$ since the elements, Δx_{it} and Δe_{it}, are orthogonal by assumption,

$Q_{22} = \left[(2\pi k)\int_0^1 cos^2(2\pi kr)dr\right]^{-1}\left[W(1) + (2\pi k)\int_0^1 sin(2\pi kr)W(r)dr\right]$,

$Q_{23} = 0$ since $\sum_{t=2}^{T}\Delta \sin\left(\frac{2\pi kt}{T}\right)\Delta \cos\left(\frac{2\pi kt}{T}\right) = 0$,

$Q_{24} = o_p(1)$ since $T^{-1}\sum_{t=2}^{T}\Delta \sin\left(\frac{2\pi kt}{T}\right)\Delta x_{\text{it}} = o_p(1)$,

$Q_{33} = \left[\int_0^1 sin^2(2\pi kr)dr\right]^{-1}\left[\int_0^1 cos(2\pi kr)W(r)dr\right]$,

$Q_{34} = o_p(1)$ since $T^{-1}\sum_{t=2}^{T}\Delta \cos\left(\frac{2\pi kt}{T}\right)\Delta x_{\text{it}} = o_p(1)$,

$Q_{44} = o_p(1)$ since $T^{-1}\sum_{t=2}^{T}\Delta x_{it}'\Delta x_{it} \to \Sigma_{w_i}$ and Δx_{it} and Δe_{it}, are orthogonal by assumption, which we obtain using the results in equations (3.12) and (A.5) - (A.7) of Enders and Lee (2012, p. 594-595) and the results in Theorem 2.1 in Chapter 2.

(ii) We write the second term on the right-hand side of (3.22) as

$$\left(\frac{1}{T}X_i'X_i\right)^{-1}\frac{1}{\sqrt{T}}X_i'\left[fH(H^{-1}\pi_i - \hat{\pi}_i) + (fH - \hat{f})\pi_i\right]$$

$$= -\left(\frac{1}{T}X_i'X_i\right)^{-1}\frac{1}{\sqrt{T}}X_i'fHa_i - \left(\frac{1}{T}X_i'X_i\right)^{-1}\frac{1}{\sqrt{T}}X_i'\epsilon\hat{\pi}_i \tag{3.24}$$

where $\epsilon = (\epsilon_2, .., \epsilon_T)'$. The first term in (3.24) is $o_p(1)$ since $a_i = o_p(1)$ and $\frac{1}{\sqrt{T}}X_i'f = O_p(1)$; see Bai and Carrion-i-Silvestre (2009, p. 497). Also, we have

$$\frac{1}{\sqrt{T}}X_i'\epsilon = = \begin{bmatrix} \frac{1}{\sqrt{T}}\sum_{t=2}^{T}\epsilon_t \\ \frac{1}{\sqrt{T}}\sum_{t=2}^{T}\Delta\, sin(2\pi t/T)\epsilon_t \\ \frac{1}{\sqrt{T}}\sum_{t=2}^{T}\Delta\, cos(2\pi t/T)\epsilon_t \\ \frac{1}{\sqrt{T}}\sum_{t=2}^{T}\Delta x_t\epsilon_t \end{bmatrix} = \begin{bmatrix} o_p(1) \\ o_p(1) \\ o_p(1) \\ o_p(1) \end{bmatrix}$$

by using the result of Bai and Ng (2004),

$$\underset{1\le m\le T}{Max}\,(N,T)\frac{1}{\sqrt{T}}\sum\left\|\hat{f}_t - Hf_t\right\| = O_p(N^{-1}) + O_p\left(T^{-3/4}\right).$$

The second term in (3.24) is $o_p(1)$. Thus, we have

$$\left(\frac{1}{T}X_i'X_i\right)^{-1}\frac{1}{\sqrt{T}}X_i'\left(f\pi_i - \hat{f}\hat{\pi}_i\right) = \mathrm{o}_p(1). \tag{3.25}$$

(d) The fourth term in (3.17) is associated with the issue of whether the frequency parameters can be estimated consistently. When we use a fixed frequency k or a fixed number of cumulative frequencies, X_{it} is pre-determined. When we estimate the frequency parameter using the usual information criteria, we have $X_{it} - \hat{X}_{it} = o_p(1)$. When we allow for heterogeneous frequency parameters, we also have $X_{it} - \hat{X}_{it} = o_p(1)$. Thus, we can maintain

$$\frac{1}{\sqrt{T}}\sum_{s=2}^{t}\left(X_{is}' - \hat{X}_{is}'\right)\hat{\delta}_i = o_p(1). \tag{3.26}$$

Therefore, using the results in (3.18), (3.20), (3.23), (3.25), and (3.26), we can show

$$\frac{1}{\sqrt{T}}\sum_{s=2}^{t}\Delta\hat{e}_{is} = \frac{1}{\sqrt{T}}\tilde{S}_{[rT]} \rightarrow \sigma V(r) \tag{3.27}$$

where

$$V(r) = W(r) - rW(1) - \left[(2\pi k)\int_0^1 cos^2(2\pi kr)dr\right]^{-1}$$
$$\left[W(1) + (2\pi k)\int_0^1 sin(2\pi kr)W(r)dr\right]sin(2\pi kr)$$
$$-\left[\int_0^1 sin^2(2\pi kr)dr\right]^{-1}\left[\int_0^1 cos(2\pi kr)W(r)dr\right]cos(2\pi kr).$$

For the second step of the regression in (3.11), we define $\Delta Z = (\Delta Z_2, .., \Delta Z_T)$, with $\Delta Z_t = (1, \Delta\, cos(2\pi kt/T), \Delta\, sin(2\pi kt/T))'$, and $\tilde{S}_i = (\tilde{S}_{i1}, .., \tilde{S}_{i,T-1})'$. Then, we have

$$T^{-2}\tilde{S}_i'M_{\Delta Z}\tilde{S}_i \rightarrow {\sigma_i}^2\int_0^1 \underline{V}(r)^2dr \tag{3.28}$$

where $\underline{V}(r)$ is the projection of the process $V(r)$ onto the orthogonal complement of the space spanned by $dZ = (1, d\,cos(2\pi kr), d\,sin(2\pi kr))'$, with $r \in [0,1]$. Also, we have

$$T^{-1}\tilde{S}_i'M_{\Delta Z}\Delta\tilde{S}_i \rightarrow -\frac{1}{2}\sigma_{\varepsilon_i}^2\int_0^1 \underline{V}(r)^2dr\,. \tag{3.29}$$

The desired result holds with (3.28) and (3.29).

Remark 3.1. Theorem 3.1 indicates that the univariate LM-based cointegration test statistic is identical to the univariate LM-based unit root test suggested by Enders and Lee (2012). Hence, the asymptotic distribution of $\tilde{\tau}_{FLM}$ does not depend on neither the number of regressors nor the factor structure, and it is free of the estimated factors and the factor loadings. This gives us the opportunity to control for the cross-correlations and the smooth breaks at the same time without generating new critical values. Nonetheless, in order to obtain reliable results, we use the response surface functions to generate critical values and corresponding *p*-values for finite samples.

Remark 3.2. To control for the effects of autocorrelations, we use the parametric method, since Westerlund and Edgerton (2007b) suggest that this method works best. Hence, we adopt our testing regression as follows

$$\Delta\tilde{S}_{it} = \phi\tilde{S}_{it-1} + constant + \sum_{k=1}^{m} c_{1ik}\Delta sin\left(\frac{2\pi kt}{T}\right) + \sum_{k=1}^{m} c_{2ik}\Delta cos\left(\frac{2\pi kt}{T}\right) + \sum_{j=1}^{p_i} \theta_{ij}\Delta\tilde{S}_{it-j} + error \quad (3.30)$$

with p_i showing the number of lags for each cross-section. In the parametric approach, we need to determine the number of lags. Unless the optimal value of p_i is known, one can use an information criterion, such as the Akaike (AIC) or the Schwarz (BIC) or can chose to be the largest integer less than $4(T/100)^{2/9}$ as suggested by Westerlund and Edgerton (2007b).

Remark 3.3. Our testing process can be extended by employing estimated frequencies. Note that Nazlioglu et al. (2022) suggest using a fixed small value of m, since using a large number of cumulative frequencies can lead to overfitting and a loss of power. Nonetheless, the number of the cumulative frequencies in (3.6) or the optimal value of the representative frequency in (3.7) can be estimated. Enders and Lee (2012) and Nazlioglu et al. (2022) utilize the *F* test and they determine the optimal frequency by checking the joint significance of Fourier coefficients.

Remark 3.4. In the PANIC approach, we need to estimate the number of factors, r. Under the assumption that r is the same for cross-sections, it can be consistently estimated by using the following criterion function proposed by Bai and Ng (2002).

$$IC(s) = \ln\left(V\left(s, \hat{F}^s\right)\right) + s \cdot g(N,T) \quad (3.31)$$

where $V\left(s, \hat{F}^s\right)$ is obtained by using estimated factors and factor loadings in $V\left(s, \hat{F}^s\right) = \min_{\pi} \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(v_{it} - \pi_i^{s\prime}F_t^s\right)^2$ and the penalty function, g, in Bai and Ng (2002) depends

only on N and T, given by $C_{NT} = \min(\sqrt{N}, \sqrt{T})$. The s defined by minimizing $IC(s)$ is the estimate of r.

3.3.1. Panel statistics

One of the main motivations for the developing panel cointegration tests is the power increases with an increase in the number of cross-sections. To this end, we propose the panel cointegration tests in this section. The panel statistic is easily obtained by taking the average of individual statistics, which is given by

$$\overline{\tilde{\tau}_{FLM}} = N^{-1} \sum_{i=1}^{N} \tilde{\tau}_{FLM_i}. \tag{3.32}$$

The asymptotic distribution of $\overline{\tilde{\tau}_{FLM}}$ can simply be derived as the average of limiting distributions of individual statistics $\tilde{\tau}_{FLM_i}$. Theorem 3.1 shows that the asymptotic distribution of τ_{FLM_i} only depends on Fourier frequency as $T \to \infty$. By the Lindberg-Levy central limit theorem under the null hypothesis as first $T \to \infty$ and then $N \to \infty$, $\overline{\tilde{\tau}_{FLM}}$ converges to the standard normal distribution with the mean μ and the variance ζ^2. Hence, the standardized panel statistic can be shown as

$$Z_{\tau}(m) = \frac{\sqrt{N}(\overline{\tilde{\tau}_{FLM}} - \mu)}{\zeta} \sim N(0,1). \tag{3.33}$$

Remark 3.5: The mean and variance of $Z_{\tau}(m)$ statistic can be obtained by conducting a Monte Carlo simulation of the limiting distribution of test statistic if no closed-form expression exists (Westerlund, 2012). The asymptotic moments are simulated for $T = 1{,}000$ with 50,000 replications for the different values of m since the asymptotic distribution of the individual statistic only depends on the Fourier frequency. Table 3.1 reports the asymptotic moments.

Table 3.1. Asymptotic moments

m	μ	ζ^2
1	-2.954	0.371
2	-3.708	0.384
3	-4.336	0.390

The DGP is given by $y_t = \alpha_0 + \alpha_1 t + \sum_{k=1}^{m} \gamma_{1k} sin\left(\frac{2\pi kt}{T}\right) + \sum_{k=1}^{m} \gamma_{2k} cos\left(\frac{2\pi kt}{T}\right) + \beta x_t + \pi F_t + e_t$, $x_t = x_{t-1} + w_t$ and $e_t = e_{t-1} + \varepsilon_t$ with $F_t, w_t, \varepsilon_t \sim N(0,1)$ and $\alpha_0, \alpha_1, \gamma_{1k}, \gamma_{2k}, \beta, \pi \sim$U[0,1] that $U[\cdot]$ denotes the uniform distribution.

Since standardized statistics require evaluating the mean and variance, either asymptotically or in finite-sample simulations, such tests can be less convenient from a practical standpoint (Bai and Carrion-i-Silvestre, 2009). Moreover, the homogenous Fourier frequency, m, across cross-sections is assumed for that $Z_\tau(m)$ converges to the standard normal distribution. It is more convenient to use the *p*-values for panel statistics to avoid moment calculation and to consider heterogeneous Fourier frequencies. Hence, it is simple to obtain the panel statistic by combining the significance levels of individual statistics proposed by Fisher (1932) as advocated by Maddala and Wu (1999) and Choi (2001). The Fisher-type test is

$$P(m) = \ -2 \ \sum_{i=1}^{N} ln\tilde{p}_i \rightarrow \chi^2_{2N} \tag{3.34}$$

where $\tilde{p}_i$ is the *p*-value of the test statistic $\tilde{\tau}_{FLM_i}$. The *P* statistic is designed for panels with fixed N.

Theorem 3.2: *If $e_{i,t}$ in equation (3.2) is independent across i and as $T \rightarrow \infty$ and N is fixed, P has a χ^2 distribution with $2N$ degrees of freedom under the null hypothesis of $H_0: \phi_i = 0$ for all i. It diverges to $+\infty$ under the alternative hypothesis of $H_1: \phi_i < 0$ for some i.*

Proof: If test statistics are continuous, the significance levels $\tilde{p}_i$ $(i = 1,2, \dots, N)$ are independent uniform $(0,1)$ variables as $T \rightarrow \infty$, and $-2\ln(\tilde{p}_i)$ has a χ^2 distribution with two degrees of freedom. Using the additive property of χ^2 variables, we obtain $P \sim \chi^2_{2N}$ for fixed N under the null hypothesis. This is valid since the asymptotic distribution of $\tilde{\tau}_{FLM_i}$ does not depend on the factor structure. Hence, the theorem holds as $T \rightarrow \infty$ and N is fixed. Under the alternative hypothesis, $\tilde{p}_i$ converges to zero in probability as $T \rightarrow \infty$; thus P diverges to $+\infty$.

As shown in Choi (2001), P statistic is unusable for the panel with a large N, and it can be modified for large N as follows,

$$P_M(m) = \frac{-2\sum_{i=1}^{N} ln\tilde{p}_i \quad - 2N}{\sqrt{4N}} \rightarrow N(0,1). \tag{3.35}$$

The panel statistic in (3.35) has a standard normal distribution under the assumption that $N \rightarrow \infty$. However, this assumption can possibly be extended. We can show that the modified Fisher statistic, P_M in (3.35), can converge to a standard normal distribution when

$T\rightarrow\infty$, followed by $N\rightarrow\infty$, or as both T, $N\rightarrow\infty$ simultaneously. Theorem 3.3 shows this result.

Theorem 3.3: *If $e_{i,t}$ in equation (3.2) is independent across i, by applying the Lindeberg-Lévy central limit theorem to (3.35) as $T\rightarrow\infty$ and followed $N\rightarrow\infty$ or as $T,N\rightarrow\infty$ jointly with $N/T\rightarrow 0$, P_M has a standard normal distribution under the null hypothesis of $H_0:\phi_i=0$ for all i. If $N_1/N\rightarrow$ c (a fixed constant) as $N\rightarrow\infty$, it diverges to $+\infty$ under the alternative hypothesis of $H_1:\phi_i<0$ for some i.*

Proof: It is clear that $E[-2\ln(\tilde{p}_i)]=2$ and $var[-2\ln(\tilde{p}_i)]=4$. By using these moments, the modified test statistic can be written as

$$P_M=\sum_{i=1}^{N}\left(\frac{-2\ln(\tilde{p}_i)-E[-2\ln(\tilde{p}_i)]}{\sqrt{var[-2\ln(\tilde{p}_i)]}}\right)=\frac{1}{\sqrt{N}}\sum_{i=1}^{N}(-\ln(\tilde{p}_i)-1) \tag{3.36}$$

Applying the Lindeberg-Lévy central theorem as $T\rightarrow\infty$ and then $N\rightarrow\infty$, we have $P_M\sim N(0,1)$ under the null hypothesis. For the case where $T,N\rightarrow\infty$ simultaneously, with $N/T\rightarrow 0$,

$$\sum_{i=1}^{N}\ln\tilde{p}_i=\ln\left(\prod_{i=1}^{N}\left(\tilde{p}_i+O_p\left(min\left[\frac{1}{\sqrt{N}},\frac{1}{\sqrt{T}}\right]\right)\right)\right) \tag{3.37}$$

$$=\ln\left(\prod_{i=1}^{N}\tilde{p}_i+O_p\left(min\left[\frac{1}{\sqrt{N}},\frac{1}{\sqrt{T}}\right]\right)\right), \tag{3.38}$$

which, in the limit, converges to $\ln(\prod_{i=1}^{N}\tilde{p}_i)=\sum_{i=1}^{N}\ln\tilde{p}_i$. This implies that $P_M\sim N(0,1)$ as $T,N\rightarrow\infty$ with $N/T\rightarrow 0$.

To show the consistency of P_M statistic by following Choi (2001), let without loss of generality N_1 cross-section have cointegration, and the rest $(N-N_1)$ have no-cointegration under the null hypothesis. We have under the alternative hypothesis that

$$P_M=\sqrt{N}\left\{\frac{1}{N}\sum_{i=1}^{N}(-\ln(\tilde{p}_i))-1\right\} \tag{3.39}$$

$$=\sqrt{N}\left\{\frac{N_1}{N}\frac{1}{N_1}\sum_{i=1}^{N}(-\ln(\tilde{p}_i)-\frac{N-N_1}{N}\frac{1}{N-N_1}\sum_{i=N_1+1}^{N}\ln(\tilde{p}_i)-1\right\} \tag{3.40}$$

$$=\sqrt{N}\left\{\frac{N_1}{N}A_{N_1}-\frac{N-N_1}{N}\frac{1}{N-N_1}B_{N_1}-1\right\}. \tag{3.41}$$

Under the alternative hypothesis, $-\ln(\tilde{p}_i) \to \infty \ \forall i = 1, \dots, N_1$ as $T \to \infty$, implying that $A_{N_0} \geq \min\{-\ln(\tilde{p}_1), \dots, -\ln(\tilde{p}_{N_o})\} = \infty$. Since $E[\ln(\tilde{p}_i)] = -1$, $B_{N_1} \to -1$ by the law of large numbers. If $\frac{N_1}{N} = c$ (a constant) as $N \to \infty$, we have $P_M \to \infty$ as $T \to \infty$ and then $N \to \infty$ under the alternative hypothesis.

3.3.2. Response surfaces to approximate *p*-values

The *p*-value of $\tilde{\tau}_{FLM}$ for each cross-sectional unit is required to obtain the Fisher-type panel statistics based on the combination of *p*-values. Thus, we estimate a response surface function to get the *p*-value corresponding to $\tilde{\tau}_{FLM}$. In that respect, we specify the data generating process as $y_t = d(t) + x_t'\beta + \pi' F_t + e_t$, $x_t = x_{t-1} + w_t$ and $e_t = \rho e_{t-1} + \varepsilon_t$ where $F_t \sim N(0,1), w_t \sim N(0,1)$, $\varepsilon_t \sim N(0,1)$, $\pi \sim U[0,1]$, and $d(t)$ is described in (6), wherein all of the parameters are taken from $U[0,1]$, and where $U[\cdot]$ denotes the uniform distribution. Since we follow the same procedures with response surface estimates defined in Chapter 2, we report the estimates of response surface regression with/without lags at the significance levels of 0.01, 0.05, and 0.10 for the different combinations of the cumulative Fourier frequencies in Table 7.11 and Table 7.12 in Appendix B.

3.4. Small sample simulations

In this section, the small sample properties of the proposed tests are examined through Monte Carlo simulations. We focus on the performance of the panel statistics in the presence of various types of breaks. The general form of DGP is given by

$$y_{it} = d_i(t) + x_{it}'\beta_i + v_{it} \tag{3.42}$$

$$v_{it} = \pi_i' F_t + e_{it} \tag{3.43}$$

$$\Delta x_{it} = w_{it} \tag{3.44}$$

$$e_{it} = \rho_i e_{it-1} + \varepsilon_{it} \tag{3.45}$$

where $w_{it} \sim N(0,1)$, $\beta_i = 1$ for all i, $\rho_i = 1 \ \forall i$ for size analysis and $\rho_i = 0.9 \ \forall i$ for power analysis. Following Westerlund and Edgerton (2008), we consider two cases of disturbance ε_{it}. In the first, ε_{it} follows a first-order moving average (MA) process, given by $\varepsilon_{it} = \xi_{it} + \theta \xi_{t-1}$ with $\theta = \{0.3, -0.3\}$. In the second case, ε_{it} follows a first-order autoregressive (AR) process as $\varepsilon_{it} = \varphi \varepsilon_{it-1} + \xi_{it}$ with $\varphi = \{0.3, -0.3\}$. Here, $\xi_{it} \sim N(0,1)$ for both cases. For the factor structure, we follow Nazlioglu et al. (2022) and describe the common factor as

$F_{jt} = \psi F_{jt-1} + u_{jt}$, where $u_{jt} \sim N(0,1)$, $\psi = \{0.9, 1\}$ with the factor loadings $\pi_i \sim N(0,1)$, and $j = 1, \dots, r$. Hence, we allow the presence of multiple dynamic common factors that are assumed to be either stationary, i.e., $\psi = \{0.9\}$, or unit root processes, i.e., $\psi = \{1\}$. We set $r = 2$ in the simulations and estimate the number of factors using the information criterion (ICp) developed by Bai and Ng (2002) with the maximum number of common factors $r_{max} = 5$. In all simulations, we use one regressor since the distribution of test statistic does not depend on the number of regressors.[10] Moreover, we compare the proposed tests with sharp break LM-based tests, Z_τ and Z_ϕ of Westerlund and Edgerton (2008, hereafter WE) for all cases. Additionally, we also regard the estimated frequency situation. For the $P(\hat{m})$ and $P_M(\hat{m})$ test with estimated frequency, we choose the maximum number of cumulative frequencies, m_{max}, is 3.[11] In all scenarios, we consider $N = \{20, 40\}$ units and $T = \{100, 200\}$ time observations. The simulations are conducted using 5,000 replications at the 5% level of significance.

To analyze the performance of the panel statistics for various types of breaks, we consider four experiments. We define $d_i = \{\} \; \forall i$ for no break experiment. In the sharp break experiment, we follow WE (2008) and describe $d_i(t) = \alpha_{i1} D_{it}$ where $\alpha_{i1} = 5$, $D_{it} = 1$ if $t > 0.5T$ and zero otherwise for all i. Then, we proceed with a smooth break experiment and indicate $d_i(t) = \alpha_{i1} \sin\left(\frac{2\pi k_i t}{T}\right) + \alpha_{i2} \cos\left(\frac{2\pi k_i t}{T}\right)$ where $\alpha_{i1} = \alpha_{i2} = 3$, $k_i = 1$ for all i. Moreover, we also consider the case in which no breaks, sharp breaks, and smooth breaks occur together. Thus far, we have assumed that cross-sections in the panel have no breaks, sharp breaks, or smooth breaks. However, it is quite reasonable to imagine a situation in which some cross section units experienced sharp breaks while others may be characterized by smooth breaks. This more realistic scenario motivates us to consider the DGP characterized by no breaks, sharp breaks, and smooth breaks. For the DGP with different forms of breaks, we generate *N* cross-sections and consider the case where 2*N*/10 individuals experience no breaks, 4*N*/10 experience sharp breaks, and 4*N*/10 experience smooth breaks.

[10] To perform a robustness check, we consider the different number of regressors, *h*, and the different maximum number of common factors, r_{max}. We obtain the size and power properties similar to those reported in Table 3.2 and 3.3. The simulation results for $h = (2,3,4,5)'$ and $r_{max} = (2,3,4,6,7)'$ are available in Tables 7.13-7.20 and Tables 7.21-7.30 in the Appendix B, respectively.

[11] We estimate *m* sequentially by checking the joint significance of the Fourier coefficients at the 10% significance level of the F-test, starting from a maximum value of, say, $j = m_{max}$. If these coefficients are insignificant, we use fewer cumulative frequencies with $j = (m_{max} - 1)$ and check the significance of the corresponding coefficients.

Table 3.2 presents the size properties of the LM-based panel cointegration test statistics for stationary common factors. The $P(m)$ and $P_M(m)$ statistics with $m = 1$ and estimated frequency are very close to 5% nominal size, regardless of the type of breaks and autocorrelation process. However, the $P(m)$ and $P_M(m)$ statistics with $m = 2$ are undersized when φ is negative and mild size distortions are observed for $Z_\tau(m)$ with $m = 2$. Further, $Z_\tau(m)$ with $m = 1$ have fairly good size except for positive values of φ. On the other hand, Z_τ, developed by WE (2008), shows mild size distortions for all autocorrelation processes for no break and sharp break. Z_ϕ, proposed by WE (2008), is undersized for positive values of φ and undersized for negative values of θ in no break and sharp break experiments. When we consider smooth or mixed breaks in DGP, Z_τ and Z_ϕ are undersized for all autocorrelation processes.

Table 3.3 shows the power properties of the LM-based panel cointegration test statistics for stationary common factors. Panel statistics with fixed frequency or estimated frequency have good power properties especially for smooth and mixed breaks. However, as reported by Becker et al. (2006), the power of panel statistics decreases as *m* increases since a high number of frequency terms decreases the degrees of freedom and can lead to an over-fitting problem. Additionally, the power of panel statistics rises as N and T grow, which is presumably reflecting the test's consistency. While test statistics, proposed by WE (2008), are powerful, especially for sharp breaks, they have very low power when the presence of smooth or mixed breaks.

Tables 3.4 and 3.5 report the size and power properties of the LM-based panel cointegration test statistics for non-stationary common factors. These results are similar to those presented in Tables 3.2 and 3.3 in that they indicate reasonably good size and power of our tests across all of the break types considered. This further reinforces our assertion that the panel statistics do not depend on the factor structure or the type of breaks.

Table 3.2. Size properties of panel statistics for stationary common factors ($\psi = 0.9$)

B. Type	θ	φ	N	T	$P(1)$	$P_M(1)$	$P(2)$	$P_M(2)$	$P(\hat{m}_i)$	$P_M(\hat{m}_i)$	$Z_\tau(1)$	$Z_\tau(2)$	Z_τ	Z_ϕ
No break	0	0	20	100	0.049	0.060	0.044	0.050	0.054	0.061	0.088	0.117	0.104	0.064
			20	200	0.054	0.063	0.048	0.054	0.054	0.063	0.071	0.077	0.074	0.058
			40	100	0.044	0.051	0.030	0.036	0.052	0.061	0.085	0.124	0.118	0.064
			40	200	0.046	0.055	0.032	0.037	0.053	0.061	0.073	0.072	0.091	0.067
	0.3	0	20	100	0.041	0.047	0.037	0.046	0.039	0.048	0.058	0.104	0.091	0.047
			20	200	0.038	0.045	0.046	0.052	0.057	0.067	0.048	0.075	0.068	0.046
			40	100	0.032	0.039	0.032	0.039	0.040	0.045	0.045	0.119	0.086	0.048
			40	200	0.039	0.047	0.041	0.045	0.054	0.062	0.051	0.085	0.071	0.032
	-0.3	0	20	100	0.033	0.043	0.029	0.037	0.030	0.037	0.046	0.070	0.085	0.202
			20	200	0.047	0.055	0.046	0.057	0.046	0.054	0.063	0.072	0.072	0.219
			40	100	0.020	0.024	0.021	0.026	0.021	0.025	0.029	0.078	0.092	0.283
			40	200	0.036	0.042	0.031	0.035	0.041	0.048	0.049	0.063	0.072	0.299
	0	0.3	20	100	0.057	0.067	0.049	0.057	0.070	0.082	0.100	0.138	0.078	0.003
			20	200	0.049	0.061	0.045	0.056	0.073	0.087	0.069	0.094	0.069	0.002
			40	100	0.055	0.062	0.046	0.054	0.062	0.069	0.106	0.192	0.085	0.000
			40	200	0.048	0.057	0.040	0.048	0.081	0.091	0.085	0.099	0.070	0.001
	0	-0.3	20	100	0.040	0.048	0.030	0.036	0.035	0.042	0.064	0.077	0.080	0.201
			20	200	0.047	0.056	0.039	0.048	0.047	0.055	0.061	0.065	0.076	0.242
			40	100	0.035	0.041	0.022	0.026	0.029	0.033	0.058	0.085	0.077	0.284
			40	200	0.042	0.048	0.025	0.028	0.038	0.041	0.061	0.056	0.060	0.332
Sharp	0	0	20	100	0.051	0.060	0.046	0.056	0.048	0.058	0.082	0.111	0.098	0.065
			20	200	0.042	0.052	0.037	0.044	0.060	0.071	0.060	0.066	0.080	0.065
			40	100	0.051	0.058	0.039	0.043	0.042	0.048	0.091	0.133	0.107	0.059
			40	200	0.048	0.055	0.037	0.043	0.057	0.066	0.071	0.073	0.082	0.060
	0.3	0	20	100	0.034	0.042	0.039	0.047	0.040	0.045	0.053	0.109	0.096	0.065
			20	200	0.040	0.049	0.041	0.049	0.066	0.079	0.053	0.071	0.067	0.038
			40	100	0.028	0.034	0.029	0.035	0.033	0.037	0.044	0.124	0.100	0.055
			40	200	0.034	0.039	0.037	0.041	0.051	0.057	0.051	0.082	0.084	0.036
	-0.3	0	20	100	0.024	0.030	0.025	0.032	0.035	0.042	0.037	0.066	0.087	0.227
			20	200	0.042	0.051	0.035	0.043	0.036	0.044	0.052	0.056	0.064	0.218
			40	100	0.020	0.024	0.019	0.022	0.021	0.025	0.026	0.069	0.087	0.342
			40	200	0.033	0.041	0.038	0.043	0.037	0.046	0.046	0.062	0.069	0.316
	0	0.3	20	100	0.036	0.043	0.042	0.051	0.054	0.065	0.066	0.115	0.091	0.005
			20	200	0.041	0.051	0.042	0.050	0.072	0.085	0.059	0.073	0.065	0.002
			40	100	0.032	0.037	0.026	0.029	0.051	0.058	0.067	0.139	0.097	0.000
			40	200	0.041	0.046	0.037	0.043	0.080	0.086	0.058	0.077	0.074	0.002
	0	-0.3	20	100	0.037	0.046	0.038	0.045	0.037	0.046	0.061	0.086	0.089	0.243
			20	200	0.036	0.043	0.037	0.045	0.041	0.051	0.049	0.064	0.074	0.246
			40	100	0.031	0.036	0.018	0.021	0.030	0.033	0.060	0.084	0.091	0.350
			40	200	0.031	0.037	0.030	0.033	0.038	0.045	0.051	0.055	0.072	0.359
Smooth	0	0	20	100	0.048	0.057	0.038	0.048	0.054	0.063	0.080	0.117	0.004	0.003
			20	200	0.046	0.055	0.044	0.053	0.052	0.059	0.065	0.070	0.004	0.005
			40	100	0.043	0.051	0.033	0.038	0.046	0.052	0.081	0.112	0.000	0.000
			40	200	0.043	0.052	0.035	0.042	0.052	0.061	0.073	0.071	0.001	0.002

	0.3	0	20	100	0.036	0.044	0.040	0.048	0.042	0.056	0.054	0.103	0.008	0.006
			20	200	0.034	0.040	0.043	0.054	0.050	0.060	0.048	0.074	0.013	0.009
			40	100	0.025	0.029	0.029	0.032	0.035	0.039	0.043	0.114	0.004	0.002
			40	200	0.038	0.044	0.034	0.039	0.049	0.055	0.056	0.067	0.006	0.003
	-0.3	0	20	100	0.028	0.035	0.025	0.032	0.033	0.040	0.039	0.061	0.001	0.002
			20	200	0.048	0.059	0.043	0.052	0.047	0.056	0.057	0.071	0.001	0.009
			40	100	0.018	0.023	0.016	0.017	0.023	0.028	0.028	0.065	0.000	0.000
			40	200	0.042	0.046	0.033	0.038	0.035	0.041	0.050	0.060	0.000	0.001
	0	0.3	20	100	0.054	0.064	0.053	0.062	0.066	0.077	0.094	0.150	0.010	0.000
			20	200	0.045	0.053	0.048	0.059	0.077	0.089	0.069	0.082	0.017	0.000
			40	100	0.054	0.059	0.047	0.054	0.066	0.074	0.102	0.187	0.003	0.000
			40	200	0.058	0.066	0.048	0.055	0.079	0.088	0.089	0.094	0.007	0.000
	0	-0.3	20	100	0.045	0.053	0.030	0.035	0.040	0.049	0.066	0.077	0.000	0.002
			20	200	0.040	0.049	0.038	0.044	0.050	0.058	0.058	0.063	0.000	0.012
			40	100	0.039	0.045	0.022	0.025	0.026	0.030	0.070	0.082	0.000	0.000
			40	200	0.044	0.050	0.030	0.036	0.041	0.045	0.063	0.060	0.000	0.004
Mixed	0	0	20	100	0.047	0.058	0.042	0.051	0.051	0.062	0.077	0.109	0.000	0.000
			20	200	0.050	0.059	0.047	0.055	0.058	0.072	0.066	0.077	0.001	0.002
			40	100	0.049	0.055	0.037	0.045	0.040	0.050	0.084	0.145	0.000	0.000
			40	200	0.037	0.043	0.035	0.040	0.050	0.056	0.060	0.068	0.000	0.000
	0.3	0	20	100	0.034	0.043	0.034	0.041	0.042	0.049	0.053	0.097	0.002	0.001
			20	200	0.039	0.047	0.041	0.048	0.056	0.066	0.055	0.072	0.007	0.004
			40	100	0.030	0.036	0.026	0.032	0.031	0.038	0.050	0.114	0.001	0.000
			40	200	0.036	0.040	0.039	0.044	0.054	0.060	0.051	0.081	0.003	0.001
	-0.3	0	20	100	0.029	0.033	0.022	0.028	0.028	0.032	0.037	0.061	0.000	0.000
			20	200	0.046	0.053	0.037	0.045	0.035	0.043	0.057	0.063	0.000	0.003
			40	100	0.021	0.024	0.020	0.022	0.018	0.020	0.032	0.068	0.000	0.000
			40	200	0.034	0.037	0.026	0.032	0.034	0.040	0.048	0.057	0.000	0.000
	0	0.3	20	100	0.039	0.049	0.034	0.046	0.056	0.068	0.073	0.111	0.004	0.000
			20	200	0.046	0.051	0.039	0.044	0.068	0.084	0.065	0.073	0.010	0.001
			40	100	0.036	0.043	0.032	0.040	0.053	0.061	0.085	0.145	0.000	0.000
			40	200	0.043	0.048	0.039	0.043	0.058	0.068	0.068	0.080	0.003	0.000
	0	-0.3	20	100	0.032	0.039	0.025	0.027	0.027	0.032	0.057	0.069	0.000	0.000
			20	200	0.033	0.040	0.033	0.041	0.034	0.041	0.052	0.054	0.000	0.008
			40	100	0.021	0.026	0.011	0.014	0.021	0.027	0.045	0.058	0.000	0.000
			40	200	0.037	0.042	0.023	0.028	0.031	0.037	0.049	0.051	0.000	0.001

Notes: θ refers to the MA parameter and φ refers to the AR parameter. $P(1)$, $P_M(1)$, $Z_\tau(1)$, $P(2)$, $P_M(2)$, $Z_\tau(2)$ show panel statistics obtained by fixed frequencies with m=1 and m=2, respectively. P and P_M statistics with estimated frequency are illustrated as $P(\widehat{m}_i)$ and $P_M(\widehat{m}_i)$, respectively. Z_τ and Z_ϕ tests are based on the level sharp break model developed by Westerlund and Edgerton (2008).

Table 3.3. Power properties of panel statistics for stationary common factors ($\psi = 0.9$)

B. Type	θ	φ	N	T	$P(1)$	$P_M(1)$	$P(2)$	$P_M(2)$	$P(\hat{m}_i)$	$P_M(\hat{m}_i)$	$Z_\tau(1)$	$Z_\tau(2)$	Z_τ	Z_ϕ
No break	0	0	20	100	0.584	0.621	0.219	0.250	0.525	0.563	0.807	0.479	0.994	0.985
			20	200	1.000	1.000	0.969	0.978	1.000	1.000	1.000	0.999	1.000	1.000
			40	100	0.846	0.865	0.352	0.376	0.775	0.799	0.969	0.734	1.000	1.000
			40	200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.3	0	20	100	0.277	0.308	0.118	0.133	0.263	0.300	0.414	0.273	0.959	0.967
			20	200	0.998	0.999	0.784	0.814	0.993	0.995	1.000	0.926	1.000	1.000
			40	100	0.454	0.486	0.142	0.158	0.374	0.402	0.673	0.415	0.998	0.998
			40	200	1.000	1.000	0.963	0.970	1.000	1.000	1.000	0.996	1.000	1.000
	-0.3	0	20	100	0.309	0.342	0.108	0.128	0.249	0.277	0.451	0.250	0.777	0.938
			20	200	0.997	0.997	0.838	0.863	0.994	0.995	1.000	0.957	1.000	1.000
			40	100	0.467	0.495	0.125	0.142	0.391	0.417	0.670	0.349	0.960	1.000
			40	200	1.000	1.000	0.978	0.981	0.999	0.999	1.000	0.999	1.000	1.000
	0	0.3	20	100	0.554	0.596	0.235	0.264	0.496	0.528	0.773	0.502	0.995	0.875
			20	200	1.000	1.000	0.950	0.958	1.000	1.000	1.000	0.994	1.000	1.000
			40	100	0.830	0.849	0.358	0.384	0.754	0.773	0.968	0.767	1.000	0.993
			40	200	1.000	1.000	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000
	0	-0.3	20	100	0.588	0.631	0.213	0.241	0.481	0.517	0.800	0.455	0.957	0.994
			20	200	1.000	1.000	0.975	0.981	1.000	1.000	1.000	0.997	1.000	1.000
			40	100	0.845	0.865	0.292	0.316	0.713	0.732	0.975	0.688	1.000	1.000
			40	200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Sharp	0	0	20	100	0.545	0.587	0.202	0.231	0.495	0.525	0.776	0.472	0.998	0.997
			20	200	1.000	1.000	0.958	0.966	1.000	1.000	1.000	0.993	1.000	1.000
			40	100	0.828	0.845	0.315	0.341	0.732	0.755	0.963	0.716	1.000	1.000
			40	200	1.000	1.000	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000
	0.3	0	20	100	0.266	0.295	0.100	0.119	0.232	0.261	0.417	0.252	0.976	0.978
			20	200	0.994	0.996	0.765	0.795	0.985	0.987	0.999	0.920	1.000	1.000
			40	100	0.415	0.453	0.136	0.153	0.348	0.372	0.648	0.389	0.999	1.000
			40	200	1.000	1.000	0.957	0.964	1.000	1.000	1.000	0.995	1.000	1.000
	-0.3	0	20	100	0.256	0.287	0.084	0.104	0.229	0.260	0.380	0.219	0.931	0.990
			20	200	0.990	0.991	0.785	0.816	0.934	0.942	0.998	0.915	1.000	1.000
			40	100	0.399	0.427	0.096	0.111	0.361	0.389	0.603	0.314	0.996	1.000
			40	200	1.000	1.000	0.962	0.969	1.000	1.000	1.000	0.997	1.000	1.000
	0	0.3	20	100	0.500	0.535	0.199	0.221	0.447	0.486	0.720	0.431	0.999	0.927
			20	200	1.000	1.000	0.934	0.947	0.999	0.999	1.000	0.992	1.000	1.000
			40	100	0.757	0.780	0.276	0.294	0.692	0.714	0.941	0.671	1.000	0.996
			40	200	1.000	1.000	0.998	0.998	1.000	1.000	1.000	1.000	1.000	1.000
	0	-0.3	20	100	0.532	0.572	0.201	0.223	0.411	0.449	0.746	0.428	0.989	0.999
			20	200	1.000	1.000	0.952	0.962	0.995	0.995	1.000	0.995	1.000	1.000
			40	100	0.783	0.800	0.259	0.279	0.655	0.678	0.942	0.630	1.000	1.000
			40	200	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Smooth	0	0	20	100	0.597	0.642	0.216	0.242	0.516	0.554	0.813	0.474	0.086	0.114
			20	200	1.000	1.000	0.969	0.976	1.000	1.000	1.000	0.998	0.926	0.950
			40	100	0.846	0.862	0.331	0.356	0.779	0.795	0.971	0.716	0.077	0.119
			40	200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.997	0.998

	0.3	0	20	100	0.292	0.319	0.106	0.129	0.253	0.282	0.432	0.267	0.144	0.151
			20	200	0.995	0.996	0.778	0.809	0.993	0.995	1.000	0.921	0.947	0.959
			40	100	0.442	0.473	0.128	0.144	0.375	0.399	0.659	0.406	0.169	0.183
			40	200	1.000	1.000	0.960	0.964	1.000	1.000	1.000	0.997	0.999	0.999
	-0.3	0	20	100	0.290	0.325	0.091	0.107	0.255	0.285	0.433	0.215	0.002	0.026
			20	200	0.998	0.999	0.832	0.853	0.996	0.998	1.000	0.946	0.144	0.474
			40	100	0.459	0.487	0.118	0.129	0.396	0.424	0.670	0.326	0.000	0.009
			40	200	1.000	1.000	0.982	0.987	0.999	0.999	1.000	0.999	0.165	0.664
	0	0.3	20	100	0.558	0.598	0.230	0.263	0.496	0.528	0.780	0.519	0.342	0.088
			20	200	1.000	1.000	0.951	0.959	1.000	1.000	1.000	0.996	0.999	0.974
			40	100	0.822	0.841	0.343	0.375	0.704	0.729	0.959	0.752	0.583	0.123
			40	200	1.000	1.000	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000
	0	-0.3	20	100	0.584	0.618	0.201	0.222	0.465	0.503	0.788	0.432	0.005	0.079
			20	200	1.000	1.000	0.971	0.980	1.000	1.000	1.000	0.997	0.485	0.811
			40	100	0.834	0.851	0.290	0.316	0.703	0.724	0.968	0.668	0.003	0.082
			40	200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.697	0.966
Mixed	0	0	20	100	0.547	0.586	0.200	0.229	0.441	0.482	0.761	0.460	0.013	0.022
			20	200	1.000	1.000	0.950	0.965	1.000	1.000	1.000	0.993	0.885	0.918
			40	100	0.807	0.823	0.308	0.334	0.707	0.731	0.954	0.688	0.004	0.016
			40	200	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	0.999	0.999
	0.3	0	20	100	0.260	0.284	0.093	0.111	0.229	0.256	0.401	0.248	0.033	0.044
			20	200	0.992	0.994	0.742	0.774	0.988	0.992	0.999	0.912	0.938	0.969
			40	100	0.404	0.433	0.126	0.146	0.338	0.367	0.628	0.386	0.032	0.036
			40	200	1.000	1.000	0.957	0.964	1.000	1.000	1.000	0.996	0.999	1.000
	-0.3	0	20	100	0.213	0.241	0.083	0.095	0.168	0.194	0.348	0.205	0.000	0.003
			20	200	0.988	0.991	0.757	0.786	0.979	0.982	0.998	0.905	0.031	0.306
			40	100	0.349	0.380	0.086	0.099	0.260	0.284	0.554	0.285	0.000	0.000
			40	200	1.000	1.000	0.956	0.962	1.000	1.000	1.000	0.996	0.031	0.531
	0	0.3	20	100	0.504	0.542	0.192	0.215	0.436	0.472	0.726	0.459	0.175	0.025
			20	200	1.000	1.000	0.934	0.948	1.000	1.000	1.000	0.990	1.000	0.967
			40	100	0.755	0.773	0.279	0.304	0.666	0.693	0.933	0.661	0.264	0.018
			40	200	1.000	1.000	0.997	0.998	1.000	1.000	1.000	0.999	1.000	1.000
	0	-0.3	20	100	0.470	0.512	0.146	0.167	0.348	0.379	0.691	0.367	0.000	0.015
			20	200	1.000	1.000	0.941	0.954	0.998	0.998	1.000	0.992	0.280	0.806
			40	100	0.718	0.744	0.214	0.240	0.567	0.590	0.918	0.574	0.000	0.005
			40	200	1.000	1.000	0.998	0.998	1.000	1.000	1.000	1.000	0.518	0.975

Notes: See Table 3.2.

Table 3.4. Size properties of panel statistics for non-stationary common factors ($\psi = 1$)

B. Type	θ	φ	N	T	$P(1)$	$P_M(1)$	$P(2)$	$P_M(2)$	$P(\hat{m}_i)$	$P_M(\hat{m}_i)$	$Z_\tau(1)$	$Z_\tau(2)$	Z_τ	Z_ϕ
No break	0	0	20	100	0.059	0.071	0.038	0.047	0.042	0.051	0.090	0.106	0.108	0.076
			20	200	0.047	0.060	0.042	0.051	0.047	0.056	0.076	0.083	0.083	0.064
			40	100	0.046	0.055	0.033	0.037	0.035	0.043	0.097	0.132	0.115	0.066
			40	200	0.040	0.047	0.034	0.040	0.040	0.046	0.064	0.071	0.086	0.061
	0.3	0	20	100	0.035	0.040	0.044	0.049	0.038	0.047	0.048	0.110	0.085	0.061
			20	200	0.046	0.054	0.049	0.056	0.048	0.059	0.057	0.086	0.070	0.044
			40	100	0.027	0.032	0.037	0.042	0.033	0.037	0.054	0.120	0.094	0.046
			40	200	0.034	0.040	0.037	0.045	0.041	0.050	0.052	0.078	0.072	0.032
	-0.3	0	20	100	0.036	0.043	0.029	0.035	0.027	0.033	0.048	0.082	0.094	0.199
			20	200	0.047	0.055	0.041	0.049	0.033	0.040	0.060	0.069	0.073	0.187
			40	100	0.023	0.028	0.018	0.023	0.016	0.019	0.032	0.071	0.086	0.249
			40	200	0.039	0.043	0.035	0.040	0.027	0.030	0.046	0.068	0.071	0.259
	0	0.3	20	100	0.054	0.064	0.062	0.068	0.057	0.068	0.091	0.152	0.075	0.003
			20	200	0.047	0.058	0.052	0.062	0.064	0.076	0.074	0.086	0.065	0.003
			40	100	0.046	0.051	0.049	0.056	0.060	0.066	0.096	0.178	0.076	0.001
			40	200	0.047	0.054	0.041	0.049	0.068	0.075	0.072	0.094	0.062	0.000
	0	-0.3	20	100	0.037	0.045	0.027	0.032	0.033	0.037	0.058	0.074	0.069	0.173
			20	200	0.054	0.060	0.032	0.039	0.036	0.041	0.064	0.060	0.063	0.201
			40	100	0.029	0.037	0.023	0.026	0.026	0.031	0.061	0.076	0.066	0.239
			40	200	0.036	0.042	0.030	0.035	0.025	0.031	0.051	0.052	0.051	0.284
Sharp	0	0	20	100	0.058	0.070	0.048	0.056	0.042	0.052	0.097	0.128	0.109	0.074
			20	200	0.055	0.062	0.052	0.061	0.043	0.055	0.073	0.086	0.077	0.065
			40	100	0.046	0.053	0.038	0.043	0.032	0.039	0.083	0.137	0.123	0.068
			40	200	0.046	0.052	0.036	0.043	0.041	0.046	0.068	0.079	0.087	0.067
	0.3	0	20	100	0.041	0.048	0.045	0.056	0.041	0.049	0.060	0.103	0.100	0.064
			20	200	0.051	0.058	0.046	0.054	0.055	0.063	0.064	0.091	0.082	0.050
			40	100	0.028	0.034	0.037	0.041	0.023	0.026	0.047	0.136	0.096	0.050
			40	200	0.033	0.039	0.032	0.039	0.040	0.046	0.055	0.080	0.073	0.031
	-0.3	0	20	100	0.029	0.039	0.027	0.033	0.025	0.030	0.049	0.076	0.083	0.193
			20	200	0.037	0.047	0.043	0.052	0.030	0.035	0.053	0.076	0.075	0.209
			40	100	0.022	0.027	0.019	0.025	0.015	0.017	0.033	0.079	0.082	0.276
			40	200	0.037	0.042	0.032	0.039	0.020	0.023	0.052	0.064	0.065	0.263
	0	0.3	20	100	0.054	0.063	0.046	0.055	0.055	0.064	0.094	0.136	0.084	0.006
			20	200	0.045	0.054	0.041	0.049	0.057	0.065	0.071	0.083	0.069	0.004
			40	100	0.051	0.060	0.047	0.056	0.040	0.046	0.102	0.163	0.081	0.001
			40	200	0.041	0.049	0.040	0.045	0.060	0.071	0.074	0.087	0.065	0.000
	0	-0.3	20	100	0.042	0.054	0.031	0.036	0.030	0.036	0.070	0.077	0.068	0.186
			20	200	0.044	0.051	0.032	0.039	0.026	0.033	0.055	0.062	0.062	0.200
			40	100	0.028	0.035	0.020	0.022	0.019	0.023	0.061	0.073	0.074	0.252
			40	200	0.038	0.044	0.023	0.029	0.027	0.032	0.059	0.051	0.059	0.301
Smooth	0	0	20	100	0.044	0.052	0.046	0.054	0.040	0.049	0.082	0.113	0.016	0.009
			20	200	0.052	0.060	0.051	0.059	0.042	0.051	0.066	0.089	0.000	0.000
			40	100	0.047	0.053	0.031	0.034	0.037	0.042	0.086	0.120	0.088	0.066
			40	200	0.046	0.052	0.034	0.038	0.039	0.043	0.076	0.071	0.000	0.000

	0.3	0	20	100	0.035	0.045	0.039	0.049	0.034	0.041	0.060	0.107	0.027	0.022
			20	200	0.042	0.052	0.049	0.060	0.053	0.063	0.059	0.077	0.000	0.000
			40	100	0.027	0.034	0.031	0.036	0.030	0.035	0.050	0.107	0.120	0.101
			40	200	0.036	0.044	0.039	0.043	0.042	0.050	0.054	0.084	0.000	0.000
	-0.3	0	20	100	0.026	0.029	0.023	0.026	0.021	0.024	0.039	0.054	0.000	0.000
			20	200	0.039	0.049	0.042	0.051	0.032	0.041	0.055	0.069	0.000	0.000
			40	100	0.025	0.028	0.018	0.020	0.021	0.022	0.036	0.052	0.001	0.002
			40	200	0.038	0.042	0.038	0.044	0.020	0.025	0.044	0.070	0.000	0.000
	0	0.3	20	100	0.051	0.060	0.051	0.062	0.065	0.075	0.089	0.131	0.049	0.009
			20	200	0.051	0.060	0.049	0.055	0.063	0.074	0.073	0.090	0.000	0.000
			40	100	0.046	0.053	0.037	0.041	0.047	0.057	0.089	0.156	0.199	0.048
			40	200	0.044	0.054	0.040	0.047	0.053	0.062	0.081	0.097	0.000	0.000
	0	-0.3	20	100	0.037	0.046	0.025	0.031	0.030	0.039	0.065	0.067	0.000	0.001
			20	200	0.046	0.055	0.036	0.045	0.030	0.036	0.065	0.061	0.000	0.000
			40	100	0.036	0.042	0.016	0.018	0.018	0.023	0.062	0.064	0.002	0.005
			40	200	0.035	0.042	0.026	0.030	0.026	0.032	0.059	0.054	0.000	0.000
Mixed	0	0	20	100	0.055	0.065	0.041	0.051	0.043	0.052	0.090	0.118	0.000	0.000
			20	200	0.054	0.063	0.043	0.051	0.036	0.042	0.074	0.082	0.001	0.002
			40	100	0.044	0.054	0.035	0.040	0.035	0.041	0.091	0.149	0.000	0.000
			40	200	0.044	0.049	0.032	0.036	0.040	0.047	0.069	0.071	0.000	0.000
	0.3	0	20	100	0.037	0.046	0.045	0.054	0.039	0.052	0.052	0.117	0.001	0.002
			20	200	0.049	0.060	0.043	0.054	0.049	0.059	0.064	0.086	0.004	0.001
			40	100	0.030	0.035	0.031	0.037	0.022	0.026	0.048	0.109	0.000	0.000
			40	200	0.032	0.037	0.031	0.036	0.039	0.047	0.045	0.073	0.001	0.001
	-0.3	0	20	100	0.028	0.034	0.022	0.029	0.023	0.028	0.045	0.070	0.000	0.000
			20	200	0.044	0.050	0.041	0.050	0.032	0.039	0.056	0.071	0.000	0.002
			40	100	0.019	0.021	0.020	0.022	0.014	0.017	0.032	0.072	0.000	0.000
			40	200	0.034	0.041	0.031	0.036	0.021	0.027	0.049	0.069	0.000	0.000
	0	0.3	20	100	0.042	0.050	0.042	0.052	0.055	0.065	0.071	0.129	0.002	0.000
			20	200	0.049	0.061	0.043	0.052	0.052	0.064	0.073	0.080	0.006	0.000
			40	100	0.041	0.048	0.038	0.047	0.046	0.053	0.085	0.153	0.000	0.000
			40	200	0.034	0.038	0.033	0.037	0.049	0.060	0.058	0.077	0.001	0.000
	0	-0.3	20	100	0.039	0.047	0.023	0.029	0.022	0.027	0.065	0.070	0.000	0.000
			20	200	0.042	0.050	0.027	0.032	0.033	0.040	0.056	0.051	0.000	0.006
			40	100	0.023	0.027	0.009	0.010	0.020	0.024	0.051	0.053	0.000	0.000
			40	200	0.030	0.035	0.021	0.024	0.023	0.027	0.043	0.044	0.000	0.001

Notes: See Table 3.2.

Table 3.5. Power properties of panel statistics for non-stationary common factors ($\psi = 1$)

B. Type	θ	φ	N	T	$P(1)$	$P_M(1)$	$P(2)$	$P_M(2)$	$P(\hat{m}_i)$	$P_M(\hat{m}_i)$	$Z_\tau(1)$	$Z_\tau(2)$	Z_τ	Z_ϕ
No break	0	0	20	100	0.564	0.603	0.222	0.246	0.443	0.480	0.780	0.471	0.964	0.957
			20	200	1.000	1.000	0.958	0.965	0.967	0.970	1.000	0.995	1.000	1.000
			40	100	0.824	0.844	0.342	0.366	0.645	0.666	0.955	0.712	1.000	1.000
			40	200	1.000	1.000	0.999	0.999	0.989	0.990	1.000	1.000	1.000	1.000
	0.3	0	20	100	0.271	0.306	0.116	0.129	0.229	0.258	0.411	0.267	0.902	0.907
			20	200	0.994	0.996	0.767	0.794	0.928	0.939	0.999	0.915	1.000	1.000
			40	100	0.424	0.454	0.137	0.154	0.328	0.352	0.636	0.390	0.997	0.995
			40	200	1.000	1.000	0.957	0.962	0.984	0.986	1.000	0.995	1.000	1.000
	-0.3	0	20	100	0.271	0.304	0.090	0.109	0.217	0.240	0.399	0.220	0.542	0.765
			20	200	0.992	0.995	0.784	0.814	0.889	0.893	0.999	0.924	0.998	1.000
			40	100	0.403	0.428	0.111	0.124	0.293	0.314	0.604	0.321	0.846	0.972
			40	200	1.000	1.000	0.973	0.976	0.919	0.921	1.000	0.998	1.000	1.000
	0	0.3	20	100	0.523	0.556	0.223	0.247	0.444	0.477	0.734	0.483	0.985	0.797
			20	200	1.000	1.000	0.926	0.942	0.986	0.989	1.000	0.986	1.000	1.000
			40	100	0.770	0.795	0.309	0.331	0.650	0.671	0.942	0.689	1.000	0.975
			40	200	1.000	1.000	0.999	0.999	0.999	0.999	1.000	1.000	1.000	1.000
	0	-0.3	20	100	0.500	0.538	0.178	0.198	0.398	0.431	0.719	0.393	0.844	0.959
			20	200	1.000	1.000	0.951	0.961	0.941	0.945	1.000	0.992	1.000	1.000
			40	100	0.760	0.785	0.235	0.257	0.599	0.618	0.931	0.590	0.979	1.000
			40	200	1.000	1.000	0.999	0.999	0.969	0.971	1.000	1.000	1.000	1.000
Sharp	0	0	20	100	0.544	0.581	0.224	0.253	0.407	0.436	0.752	0.467	0.975	0.969
			20	200	1.000	1.000	0.935	0.943	0.957	0.962	1.000	0.987	1.000	1.000
			40	100	0.795	0.811	0.315	0.336	0.602	0.620	0.955	0.688	1.000	1.000
			40	200	1.000	1.000	1.000	1.000	0.990	0.991	1.000	1.000	1.000	1.000
	0.3	0	20	100	0.262	0.292	0.111	0.129	0.209	0.237	0.399	0.266	0.910	0.917
			20	200	0.983	0.988	0.717	0.755	0.921	0.928	0.998	0.885	1.000	1.000
			40	100	0.412	0.442	0.142	0.157	0.285	0.308	0.617	0.398	0.998	0.998
			40	200	1.000	1.000	0.959	0.963	0.978	0.981	1.000	0.996	1.000	1.000
	-0.3	0	20	100	0.259	0.286	0.112	0.127	0.167	0.188	0.379	0.240	0.752	0.910
			20	200	0.919	0.930	0.622	0.655	0.792	0.807	0.976	0.807	0.999	1.000
			40	100	0.380	0.408	0.114	0.134	0.270	0.292	0.570	0.311	0.942	0.996
			40	200	1.000	1.000	0.960	0.967	0.901	0.905	1.000	0.994	1.000	1.000
	0	0.3	20	100	0.493	0.530	0.207	0.236	0.386	0.424	0.706	0.460	0.985	0.799
			20	200	0.999	0.999	0.905	0.920	0.982	0.984	1.000	0.982	1.000	1.000
			40	100	0.768	0.791	0.296	0.325	0.582	0.608	0.946	0.695	1.000	0.990
			40	200	1.000	1.000	0.999	1.000	0.998	0.999	1.000	1.000	1.000	1.000
	0	-0.3	20	100	0.467	0.501	0.154	0.174	0.340	0.372	0.679	0.362	0.947	0.988
			20	200	0.992	0.994	0.845	0.865	0.907	0.915	0.998	0.956	1.000	1.000
			40	100	0.732	0.758	0.226	0.248	0.533	0.558	0.919	0.576	0.997	1.000
			40	200	1.000	1.000	0.999	0.999	0.959	0.960	1.000	1.000	1.000	1.000
Smooth	0	0	20	100	0.559	0.596	0.195	0.221	0.437	0.467	0.765	0.445	0.001	0.001
			20	200	1.000	1.000	0.956	0.964	0.973	0.977	1.000	0.991	0.001	0.005
			40	100	0.820	0.839	0.289	0.313	0.639	0.662	0.954	0.672	0.003	0.003
			40	200	1.000	1.000	0.999	0.999	0.990	0.991	1.000	1.000	0.000	0.000

	0.3	0	20	100	0.278	0.309	0.107	0.126	0.209	0.239	0.408	0.249	0.002	0.003
			20	200	0.994	0.995	0.766	0.791	0.932	0.940	1.000	0.914	0.008	0.018
			40	100	0.442	0.468	0.138	0.156	0.310	0.337	0.652	0.386	0.002	0.002
			40	200	1.000	1.000	0.961	0.968	0.983	0.983	1.000	0.996	0.000	0.000
	-0.3	0	20	100	0.236	0.271	0.072	0.084	0.196	0.221	0.373	0.188	0.000	0.000
			20	200	0.995	0.996	0.774	0.801	0.886	0.893	0.999	0.918	0.000	0.000
			40	100	0.402	0.429	0.088	0.099	0.285	0.309	0.602	0.265	0.000	0.000
			40	200	1.000	1.000	0.972	0.976	0.923	0.926	1.000	0.997	0.000	0.000
	0	0.3	20	100	0.515	0.551	0.201	0.230	0.435	0.471	0.738	0.451	0.005	0.003
			20	200	0.999	0.999	0.936	0.946	0.985	0.989	1.000	0.986	0.057	0.020
			40	100	0.783	0.804	0.306	0.333	0.627	0.655	0.945	0.695	0.005	0.005
			40	200	1.000	1.000	0.998	0.998	0.999	0.999	1.000	1.000	0.008	0.002
	0	-0.3	20	100	0.483	0.518	0.157	0.178	0.389	0.431	0.711	0.357	0.000	0.000
			20	200	1.000	1.000	0.951	0.960	0.946	0.950	1.000	0.993	0.000	0.003
			40	100	0.744	0.768	0.208	0.229	0.580	0.601	0.930	0.537	0.000	0.000
			40	200	1.000	1.000	0.998	0.998	0.967	0.968	1.000	1.000	0.000	0.000
Mixed	0	0	20	100	0.501	0.534	0.201	0.224	0.402	0.433	0.709	0.433	0.007	0.037
			20	200	1.000	1.000	0.940	0.952	0.963	0.966	1.000	0.989	0.860	0.931
			40	100	0.769	0.786	0.273	0.294	0.609	0.631	0.941	0.649	0.009	0.073
			40	200	1.000	1.000	0.998	0.999	0.987	0.988	1.000	1.000	0.997	1.000
	0.3	0	20	100	0.245	0.275	0.103	0.122	0.204	0.228	0.393	0.259	0.027	0.050
			20	200	0.987	0.990	0.728	0.763	0.923	0.934	0.997	0.888	0.883	0.950
			40	100	0.386	0.418	0.129	0.143	0.287	0.310	0.601	0.369	0.022	0.062
			40	200	1.000	1.000	0.942	0.952	0.977	0.979	1.000	0.996	0.999	1.000
	-0.3	0	20	100	0.185	0.215	0.079	0.092	0.144	0.165	0.294	0.188	0.001	0.019
			20	200	0.973	0.979	0.691	0.725	0.846	0.859	0.995	0.863	0.060	0.520
			40	100	0.269	0.295	0.080	0.094	0.198	0.218	0.468	0.271	0.000	0.015
			40	200	1.000	1.000	0.922	0.931	0.912	0.916	1.000	0.990	0.121	0.869
	0	0.3	20	100	0.459	0.494	0.182	0.204	0.375	0.409	0.674	0.397	0.075	0.019
			20	200	1.000	1.000	0.909	0.926	0.983	0.986	1.000	0.985	0.996	0.932
			40	100	0.700	0.723	0.250	0.272	0.577	0.594	0.902	0.632	0.125	0.018
			40	200	1.000	1.000	0.997	0.997	0.997	0.998	1.000	1.000	1.000	1.000
	0	-0.3	20	100	0.363	0.395	0.120	0.142	0.290	0.319	0.575	0.314	0.001	0.057
			20	200	1.000	1.000	0.899	0.913	0.926	0.932	1.000	0.982	0.294	0.861
			40	100	0.595	0.622	0.164	0.179	0.451	0.475	0.843	0.491	0.000	0.064
			40	200	1.000	1.000	0.992	0.995	0.964	0.966	1.000	1.000	0.632	0.995

Notes: See Table 3.2.

3.5. Concluding remarks

In this study, we propose new cointegration tests with structural breaks for dependent panels. Regarding structural breaks and how to handle cross-sectional dependence, our tests are different from panel cointegration literature. We employ the Fourier approximation suggested by Gallant (1981) to capture multiple structural breaks. It can capture unknown multiple structural changes by using a small number of frequencies (Nazlioglu et al., 2022), and it does not require a priori information such as the number, location, or form of breaks. On the other hand, we utilize the PANIC approach of Bai and Ng (2004) to accommodate cross-sectional dependence and show that our panel cointegration tests do not depend on the factor structure or unknown forms of cross-correlations of the error terms. The asymptotic results of our tests also indicate that the distribution of test statistics only depends on the Fourier frequency and is independent of the number of regressors. Finally, Monte Carlo simulations are performed to obtain the finite-sample size and power of the suggested tests. The simulation results reveal that the proposed tests have good size and power properties for stationary or non-stationary common factors under the different types of structural breaks.

4. RE-EXAMINING THE PURCHASING POWER PARITY HYPOTHESIS FOR OECD COUNTRIES

4.1. Introduction

The doctrine of purchasing power parity (PPP) is essential to international economics. PPP, which is attributed to Cassel (1916), provides a simple determination of exchange-rate. According to this theory, an exchange rate equalizes the price of goods between countries (Bahmani-Oskooee and Hegerty, 2009). PPP is based on the law of one price (LOP), which states that after currencies are converted, each specific good should cost the same worldwide (Rogoff, 1996). However, LOP does not hold since of factors such as transport costs and trade barriers. Instead, PPP is concerned with general price levels and how they relate to the nominal exchange rate. It is proposed that exchange rates should equalize the price levels of their respective nations. Thus, PPP is defined as follows:

$$NER = \frac{P}{P^*} \tag{4.1}$$

where NER is the nominal exchange rate, P shows the domestic price level and P^* represents the foreign price level. Furthermore, the log-linear form of (4.1) can be shown as

$$\ln NER = \alpha + \beta \ln(P/P^*) \tag{4.2}$$

where the value of α and β are expected to be zero and one, respectively.

Due to the availability of long time series data and newly developed panel data techniques, the debate on PPP validity in the long-run still remains relevant. There are two common ways of testing PPP in the literature.[12] Studies in the literature examine the stationarity of the real exchange rate as the first way (Taylor, 2002; Narayan, 2005, 2006; Omay et al., 2018; Villiers and Phiri, 2022). As a second way, they investigate the possible long-run cointegration relationship between the nominal exchange rate and relative prices (Taylor, 1988; Taylor and McMahon, 1988; Salehizadeh and Taylor, 1999; Yazgan, 2003; Narayan, 2007; Narayan et al., 2009; Arize et al., 2015; Munir et al, 2018). The results obtained from these studies have been very mixed and far from convincing. There are several reasons for mixed results: (i) misspecification of the estimated test regression, (ii) the low power of univariate tests, (iii) ignoring structural breaks and (iv) assumption of cross-section independency

[12] For a comprehensive review of the literature on PPP, see Rogoff (1996), Sarno and Taylor (2002), Drine and Rault (2008) and Bahmani-Oskooee and Hegerty (2009), among others.

(Hegwood and Papell, 1998; O'Connell, 1998; Papell, 2002; Murray and Papel, 2005; Harris et al., 2005; Snaith, 2012).

In this chapter, we try to simultaneously address these issues by employing our new tests that consider structural breaks, cross-sectional dependence, and heterogeneity at the same time. The main objective of this study is to demonstrate the practical application of the new tests. To do this, we consider two datasets to testing of PPP validity. First, we use Westerlund and Edgerton (2008) dataset to compare the results. The updated dataset is then used to investigate the current validity of PPP. The results from both datasets show that the PPP is valid for 17 OECD countries.

The rest of the chapter is organized as follows. Section 2 outlines the econometric methodology. Section 3 describes the data and the empirical findings. Finally, Section 4 concludes.

4.2. Methodology

Before testing for cointegration relationship between nominal exchange rate and relative prices, we need to examine their stationarity properties. To this end, we use the panel unit root tests recently developed Bai and Carrion-i-Silvestre (2009) and Nazlioglu et al. (2022), which they consider for both cross-sectional dependency and unknown structural breaks. They defined data generating processes (DGP) as follows.

$$y_{it} = d_{it} + \pi_i' F_t + e_{it} \tag{4.3}$$

$$(I - L)F_t = C(L)u_t \tag{4.4}$$

$$(1 - \rho_i L)e_{it} = H_i(L)\varepsilon_{it} \tag{4.5}$$

with $t = 1, \dots T$ and $i = 1, \dots N$, where $C(L) = \sum_{j=0}^{\infty} C_j L^j$ and $H_i(L) = \sum_{j=0}^{\infty} \mathrm{H}_{ij} L^j$, and L is the lag operator. Here, $u_t \sim iid(0, \Sigma_u)$, $\varepsilon_{it} \sim iid(0, \Sigma_{\varepsilon_i})$ for each i, d_{it} represents the deterministic terms, and F_t is an *r*-dimensional vector of unobserved common factors, with π_i the vector of factor loadings. Bai and Carrion-i-Silvestre (2009) allow for sharp breaks in DGP and define the deterministic terms as,

$$d_{it} = \mu_i + \beta_i t + \sum_{j=1}^{l_{1i}} \theta_{ij} DU_{ijt} + \sum_{s=1}^{l_{2i}} \gamma_{ij} DT_{ist} \tag{4.6}$$

where l_{1i} and l_{2i} show number of breaks in the level and the trend, respectively. The dummy variables are denoted as $DU_{ijt} = 1$ for $t > T_{aj}^{i}$ and 0 elsewhere, and $DT_{ist} = (t - T_{bs}^{i})$ for $t > T_{bs}^{i}$ and 0 elsewhere, where T_{aj}^{i} and T_{bs}^{i} represent the break dates for the level and the trend, respectively. On the other hand, Nazlioglu et al. (2022) consider the smooth breaks in DGP and specify the deterministic terms as,

$$d_{it} = c_i + \delta_i t + \sum_{k=1}^{m_i} a_{ik} \cos\left(\frac{2\pi kt}{T}\right) + \sum_{k=1}^{m_i} b_{ik} \sin\left(\frac{2\pi kt}{T}\right) \tag{4.7}$$

where m_i shows the number of cumulative Fourier frequencies for each individual. Both panel unit root tests use the panel analysis of non-stationarity in idiosyncratic and common components (PANIC) approach of Bai and Ng (2004) to estimate unobserved common factors. Then, Bai and Carrion-i-Silvestre (2009) use modified Sargan-Bhargava (MSB) test, while Nazlioglu et al. (2022) employ t-statistic based on the Lagrange Multiplier (LM) approach to compute the individual statistic. In order to obtain panel statistics, they utilize the Fisher-type statistics, given by

$$P = -2 \sum_{i=1}^{N} ln\tilde{p}_i \rightarrow \chi_{2N}^{2} \tag{4.8}$$

and

$$P_M = \frac{-2\sum_{i=1}^{N} ln\tilde{p}_i \quad - 2N}{\sqrt{4N}} \rightarrow N(0,1) \tag{4.9}$$

where $\tilde{p}_i$ shows the *p*-value of the corresponding test statistic. The P statistics is developed for panels with fixed N, while P_M is designed for large N panels. To implement these panel statistics, the *p*-values have to approximated by simulations since the individual statistics have non-standard limiting distributions. Bai and Carrion-i-Silvestre (2009) and Nazlioglu et al. (2022) use response surface functions.

To investigate long-run cointegration relationship between nominal exchange rate and relative prices, we follow similar way as in stationarity analysis. We apply the panel cointegration test developed by Westerlund and Edgerton (2008), which allows for sharp breaks in DGP, and we employ the panel cointegration test suggested in previous chapter, which consider smooth breaks in DGP.[13]

[13] Since the previous chapter gives detailed explanation of our suggested test, we have only discussed Westerlund and Edgerton (2008)'s test in this section.

Westerlund and Edgerton (2008) extend the LM unit root testing framework of Schmidt and Phillips (1992) to the panel cointegration context with breaks and common factors. They use the following data generation process.

$$y_{it} = d'_{it}\alpha_i + x'_{it}\,\beta_i + e_{it} \tag{4.10}$$

$$x_{it} = x_{it-1} + v_{it} \tag{4.11}$$

$$e_{it} = \lambda'_i F_t + \varepsilon_{it} \tag{4.12}$$

$$F_{jt} = \phi_j F_{jt-1} + \xi_{jt} \tag{4.13}$$

$$\rho_i(L)\Delta\varepsilon_{it} = \rho_i\varepsilon_{it-1} + \eta_{it} \tag{4.14}$$

where $\rho_i(L) = 1 - \sum_{j=1}^{p_i}\rho_{ij}L^j$ is a scalar, L is the lag operator, F_t denotes r-dimensional vector of common factors F_{jt} for $j = 1, \dots, r$ and λ_i shows the corresponding vector of factor loadings. Westerlund and Edgerton (2008) defines the deterministic terms as $d_{it} = [1, t]'$ for no break model and $d_{it} = [1, t, DU_{it}]'$ for level sharp break model. Here, the level break dummy DU_{it} is $DU_{it} = 1$ if $t > Tb_i$ and zer o otherwise. The location of break that is denoted by Tb_i for individual i can be shown as $Tb_i = [\psi_i T]$ with $\psi_i \in (0,1)$.

The null hypothesis of no cointegration, $H_0: \rho_i = 0$ for all i, is tested against the alternative hypothesis of cointegration, $H_1: \rho_i < 0$ for at least some i. The equation (4.10) is estimated in first differences for each cross-section,

$$\Delta y_{it} = \Delta d'_{it}\alpha_i + \Delta x'_{it}\,\beta_i + \Delta e_{it}. \tag{4.15}$$

The principal component method of Bai and Ng (2004) is applied to Δe_{it} to obtain $\hat{F}_t$ and $\hat{\varepsilon}_{it}$.

$$\Delta\hat{e}_{it} = \lambda'_i\Delta\hat{F}_t + \Delta\hat{\varepsilon}_{it} \tag{4.16}$$

where $\hat{\varepsilon}_{it} = \sum_{t=2}^{T}\Delta\varepsilon_{it}$, $\hat{F}_t = \sum_{t=2}^{T}\Delta\hat{F}_t$, and $\hat{\lambda}_i = (T-1)^{-1}\Delta\hat{F}'\Delta\hat{e}$. Then, the LM-detrended series are

$$\hat{S}_{it} = y_{it} - \hat{\delta}_{0i} - x'_{it}\,\hat{\beta}_i - d'_{it}\hat{\alpha}_i - \hat{\lambda}'_i\hat{F}_t \tag{4.17}$$

where $\hat{\delta}_{0i} = y_{i1} - x'_{i1}\,\hat{\beta}_i - d'_{i1}\hat{\alpha}_i$. Finally, the group mean test statistics can be obtained by using the following regression,

$$\Delta\hat{S}_{it} = \alpha + \rho_i\hat{S}_{it-1} + \sum_{j=1}^{p_i}\rho_{ij}\Delta\hat{S}_{it-1} + error \tag{4.18}$$

$$\overline{LM}_\tau = \frac{1}{\mathrm{N}}\sum_{i=1}^{N}\frac{\hat{\rho}_i}{se(\hat{\rho}_i)} \tag{4.19}$$

where p_i shows the lag order and $se(\hat{\rho}_i)$ is the standard error of $\hat{\rho}_i$. The standardized panel cointegration tests statistics can be calculated as

$$Z_\tau = \frac{\sqrt{N}(\overline{LM}_\tau - \mu_\tau)}{\sigma_\tau} \sim N(0,1) \qquad (4.20)$$

where μ_τ is mean of $\overline{LM}_\tau$ and σ_τ^2 represents the variance of $\overline{LM}_\tau$. Westerlund and Edgerton (2008) obtain the asymptotic moments via Monte Carlo simulations and repot the numerical values of moments as $\mu_\tau = -8.4376$, $\sigma_\tau^2 = 25.8964$.

4.3. Data and Empirical results

4.3.1. Westerlund and Edgerton (2008) Data

In first part of our analysis, we utilize the same dataset with Westerlund and Edgerton (2008) to compare the results. The data that cover the period from the first quarter of 1973 to the last quarter of 1998 are taken from the International Financial Statistics (IFS) database of International Monetary Fund (IMF) for 17 OECD countries[14].

The dynamics of nominal exchange rate and relative prices are illustrated in Figure 4.1 and Figure 4.2, respectively. Figures also show the sharp break (dotted line) and smooth break (dashed line) functions of series. They show that the Fourier approximation superimposes the structural changes and appears to capture the long swings in the data well.

The results of the panel unit root tests are presented in Table 4.1. The results show that the null hypothesis of unit root cannot be rejected for both series and all test statistics. Thus, we can proceed to examine the long-run cointegration relationship between nominal exchange rate and relative prices, since both series are non-stationary.

[14] A list of all the countries considered in the analysis can be found in Table 4.2.

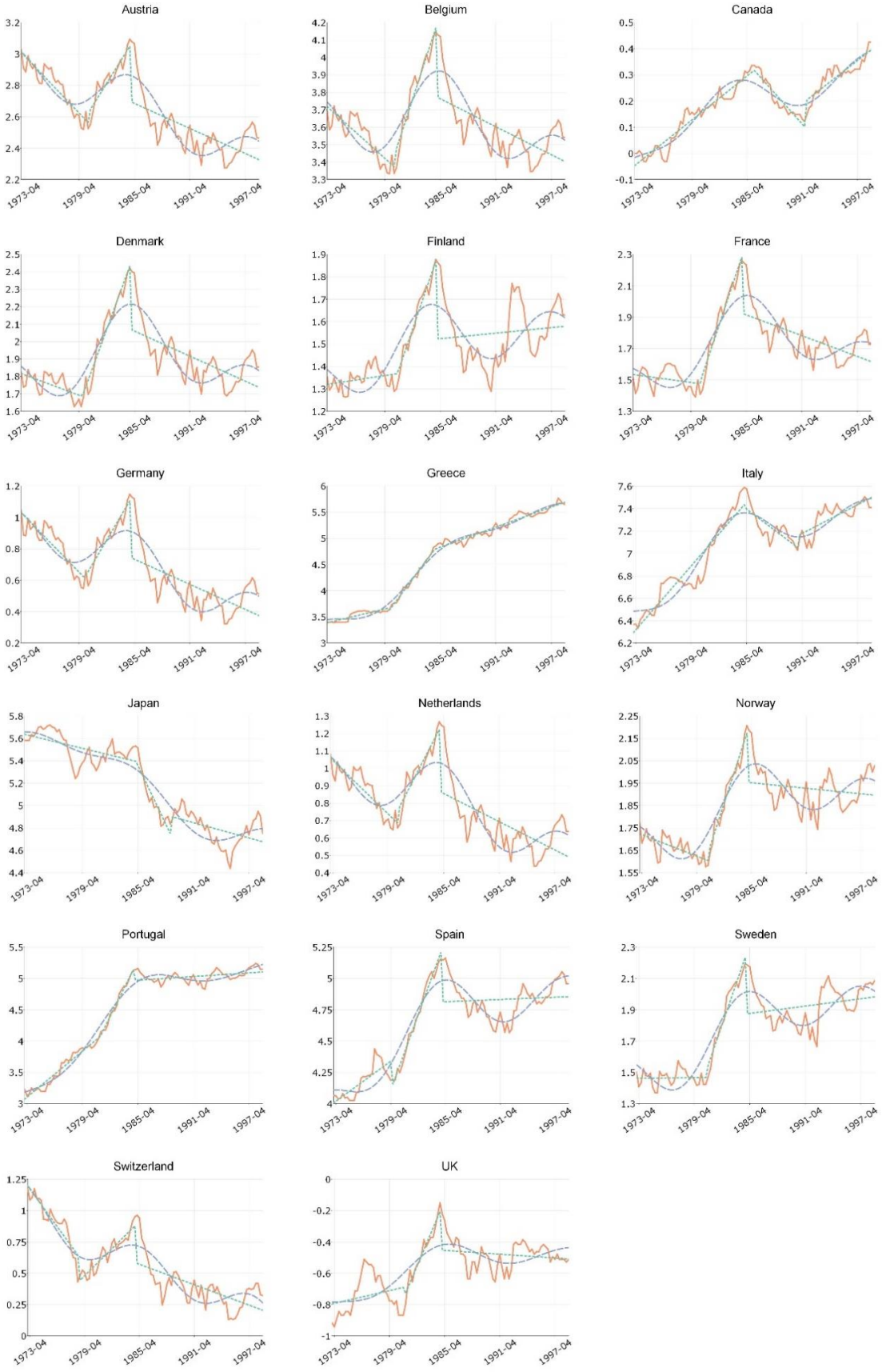


Figure 4. 2. The logarithm of nominal exchange rates with sharp (dotted line) and smooth (dashed line) breaks

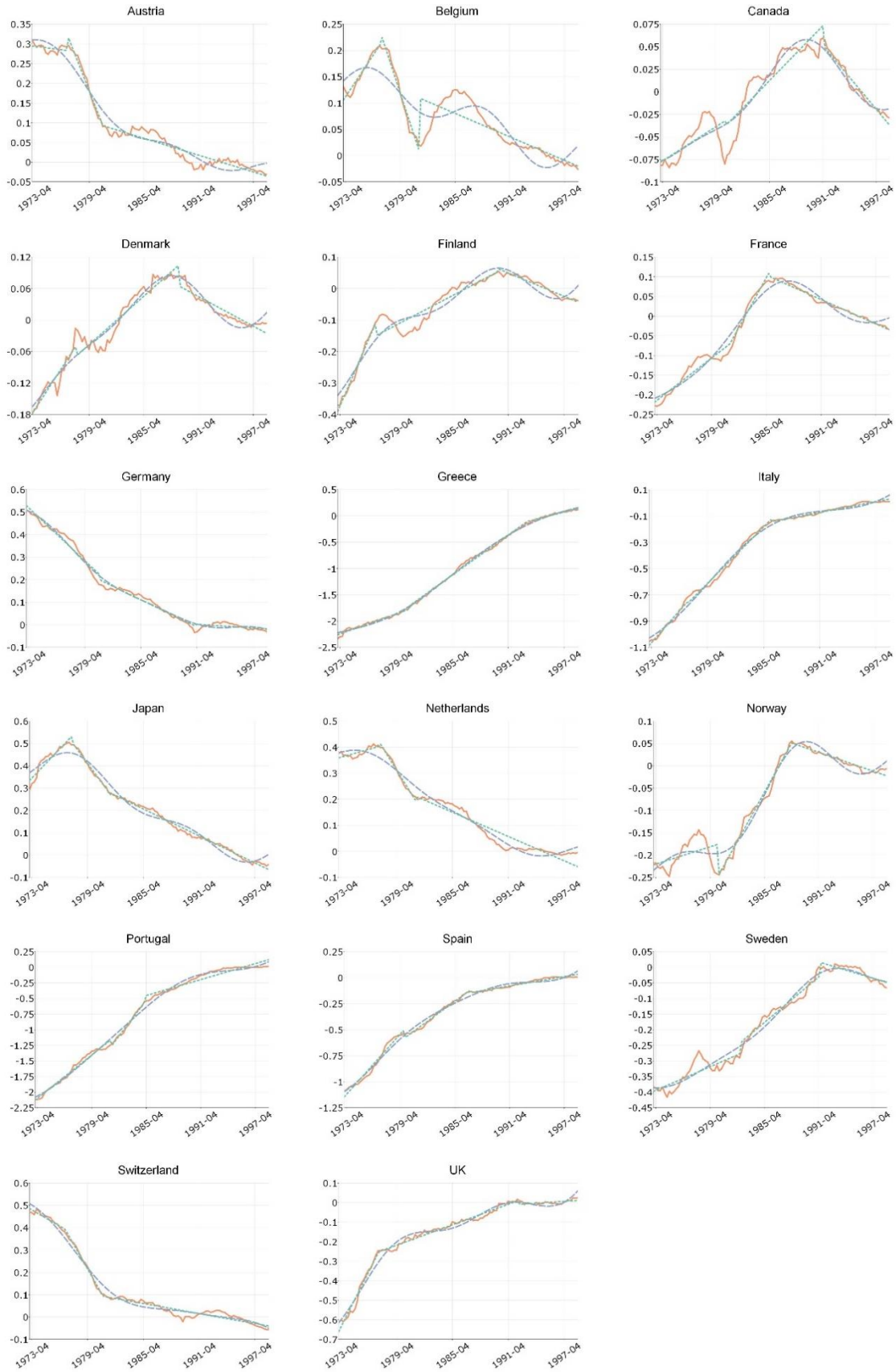


Figure 4. 3. The logarithm of relative prices with sharp (dotted line) and smooth (dashed line) breaks

Table 4.1: Results from panel unit root tests for Westerlund and Edgerton (2008) data

	PANIC with sharp breaks Bai and Carrion-i-Silvestre (2009)				PANIC with smooth breaks Nazlioglu et al. (2022)			
	$\ln NER$		$\ln(P/P^*)$		$\ln NER$		$\ln(P/P^*)$	
Tests	Statistic	*p*-value	Statistic	*p*-value	Statistic	*p*-value	Statistic	*p*-value
P	36.579	0.350	19.312	0.980	37.232	0.323	33.901	0.473
P_M	0.313	0.377	-1.781	0.963	0.392	0.348	-0.012	0.505
# Factors	5		5		1		5	

Notes: The optimal lags are determined using up to the integer value of $[12(T/100)^{0.25}]$. The maximum number of common factors is selected 5. The Fourier frequency is selected as 1.

In cointegration analysis, we follow Westerlund and Edgerton (2008) for purposes of comparison. Thus, we allow for three sharp breaks, we set the maximum number of lags to 10, and the maximum number of common factors is set to 5. For our test with estimated frequency, we select the maximum number of cumulative Fourier frequencies is 2. The results from panel cointegration tests are reported in Table 4.2. We perfectly replicate the results of Westerlund and Edgerton (2008) and provide the individual results in Table 4.2. The results show that the panel cointegration tests with smooth breaks reject the null hypothesis of no cointegration at the 1% significance level, while the panel cointegration tests with sharp breaks and no breaks do not reject the null of no cointegration. The result of our suggested test implies that there is a long-run cointegration relationship between nominal exchange rate and relative prices, which is consistent with theoretical expectations and the results of Drine and Rault (2008).

4.3.2. Updated Data

In second part of our analysis, we update the dataset, and it covers the period from the first quarter of 2002 to the third quarter of 2022. The starting date is selected the first quarter of 2002, since 12 European Union (EU) members switched from using their native currencies as legal tender to just using the euro in 2002, and 10 of these 12 EU members are included in our dataset (Britannica, 2023). The nominal exchange rates are taken from the OECD Main Economic Indicators and the consumer price indexes are obtained from the International Financial Statistics (IFS) database of International Monetary Fund (IMF) for 17 OECD countries[15].

[15] A list of all the countries considered in the analysis can be found in Table 4.4.

Table 4.2: Results from panel cointegration tests for Westerlund and Edgerton (2008) data

	Westerlund and Edgerton (2008)					This study				
	No break	Sharp Breaks				Smooth Breaks				
Country	LM_τ	LM_τ	Break Dates			$LM_\tau(1)$	p-value	$LM_\tau(\widehat{m})$	p-value	$\widehat{m}$
Austria	-2.502	-2.155	1979Q4	1981Q1	1992Q3	-5.719***	0.000	-5.810***	0.000	1
Belgium	-1.645	-2.131	1981Q1	1981Q4	1992Q3	-3.021	0.427	-3.026	0.424	1
Canada	-1.377	-2.118	1978Q2	1986Q4	1987Q4	-2.259	0.884	-2.186	0.912	1
Denmark	-0.859	-1.312	1981Q1	1987Q3	1992Q3	-3.795*	0.098	-3.850*	0.086	1
Finland	-1.936	-2.432	1987Q3	1992Q3	1992Q4	-3.693	0.124	-3.417	0.219	2
France	-1.19	-1.984	1985Q2	1990Q4	1992Q3	-3.839*	0.089	-3.961*	0.066	1
Germany	-0.971	-2.999	1987Q3	1990Q4	1992Q3	-2.706	0.631	-2.786	0.578	1
Greece	-1.418	-1.472	1982Q4	1990Q4	1992Q3	-2.857	0.532	-3.047	0.411	1
Italy	-1.35	-1.753	1975Q4	1987Q3	1992Q3	-3.135	0.359	-2.935	0.481	1
Japan	-1.511	-1.204	1985Q2	1987Q3	1995Q2	-4.747***	0.007	-5.304***	0.001	1
Netherlands	-1.625	-2.730	1981Q1	1987Q3	1990Q4	-2.804	0.566	-2.804	0.567	1
Norway	-3.021	-2.527	1981Q1	1982Q2	1992Q3	-3.619	0.146	-5.101***	0.002	1
Portugal	-1.042	-1.571	1976Q4	1983Q1	1992Q3	-4.152**	0.040	-4.145**	0.041	1
Spain	-1.455	-2.013	1977Q2	1987Q3	1992Q3	-1.787	0.990	-1.985	0.966	1
Sweden	-2.166	-1.094	1981Q1	1982Q3	1992Q3	-4.344**	0.024	-2.506	0.758	1
Switzerland	-1.266	-1.155	1978Q2	1987Q3	1992Q3	-4.102**	0.046	-3.980*	0.063	1
UK	-2.464	-2.153	1981Q1	1987Q3	1992Q3	-3.754	0.108	-5.129***	0.002	1
Panel Results	Statistic	p-value	Statistic	p-value		Statistic	p-value	Statistic	p-value	
Z_τ	2.385	0.991	0.271	0.607		-4.028***	0.000			
P						75.801***	0.000	95.041***	0.000	
P_M						5.069***	0.000	7.402***	0.000	
# Factors	5		4			5		5		

Notes: ***, **, and * indicate the rejection of the null hypothesis at 1%, 5% and 10%, respectively. The maximum number of lags and common factors are set to 10 and 5, respectively. The Fourier frequency is determined as $m = 1$ for fixed case and the maximum number of Fourier frequency is $m = 2$ for estimated case.



Figure 4. 4. The logarithm of nominal exchange rates with sharp (dotted line) and smooth (dashed line) breaks

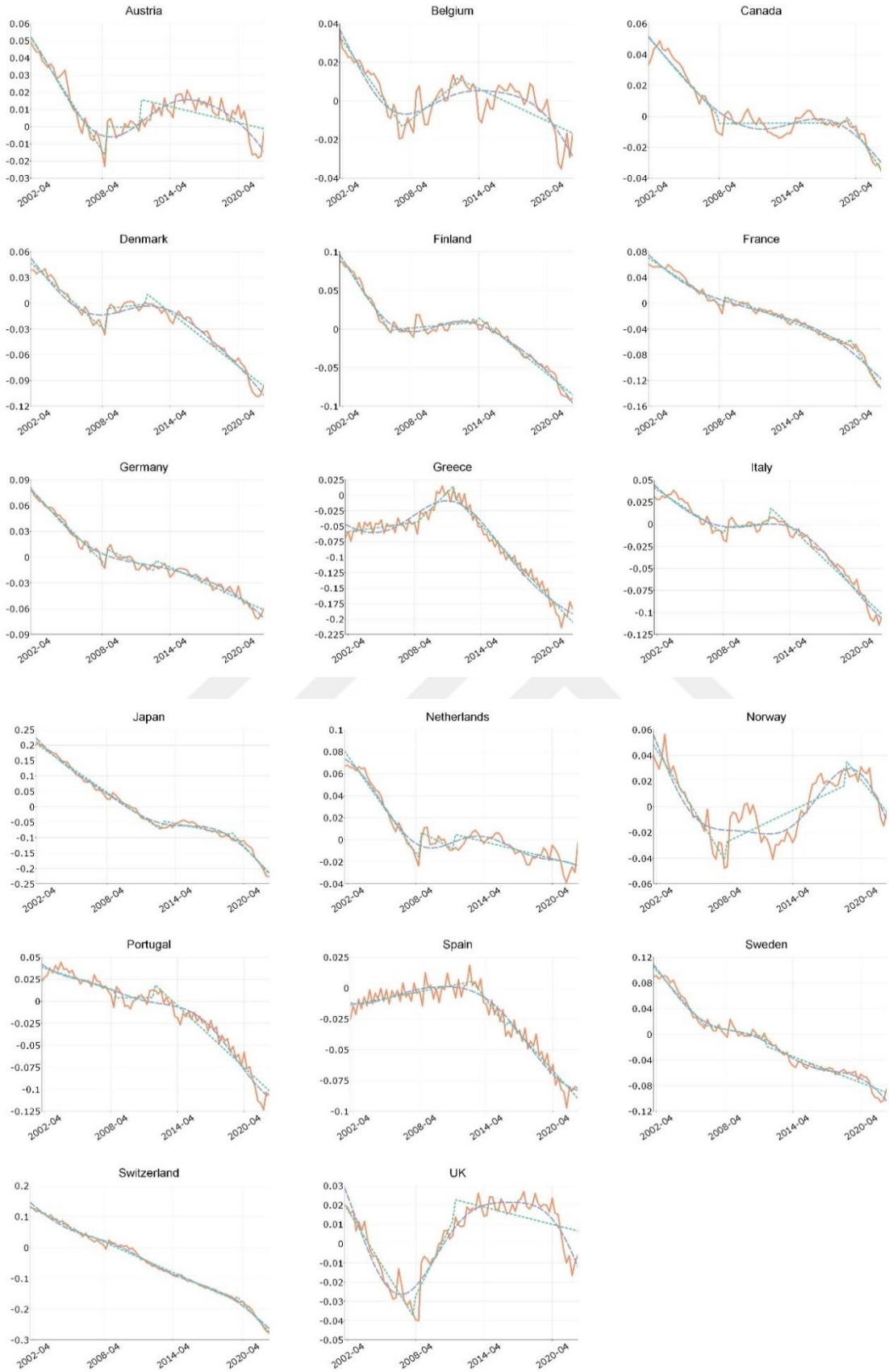


Figure 4. 5. The logarithm of relative prices with sharp (dotted line) and smooth (dashed line) breaks

As in the previous dataset, the visual inspections of the series are demonstrated in Figure 4.3 and Figure 4.4. The sharp break approximation is presented in dotted line and smooth break approximation is shown in dashed line. Figures show that the Fourier approximation superimposes the structural changes and appears to capture the long swings in the data well.

The results of the panel unit root tests are presented in Table 4.3. The results indicate that the null hypothesis of unit root cannot be rejected for both series and all test statistics. It implies the nominal exchange rate and relative prices are non-stationary. Therefore, the long-run cointegration relationship between series can be investigated.

Table 4.3: Results from Panel unit root tests for updated data

	PANIC with sharp breaks Bai and Carrion-i-Silvestre (2009)				PANIC with smooth breaks Nazlioglu et al. (2022)			
	$\ln NER$		$\ln(P/P^*)$		$\ln NER$		$\ln(P/P^*)$	
Tests	Statistic	*p*-value	Statistic	*p*-value	Statistic	*p*-value	Statistic	*p*-value
P	26.366	0.822	20.476	0.967	20.222	0.971	32.603	0.536
P_M	-0.926	0.823	-1.640	0.950	-1.671	0.953	-0.169	0.567
# Factors	2		5		5		5	

Notes: See Table 4.1.

As in the previous cointegration analysis, we allow for three sharp breaks, we set the maximum number of lags to 10, and the number of common factors is set to 5. We select the maximum number of cumulative Fourier frequencies is 2 for the estimated case of our test. The results from panel cointegration tests are reported in Table 4.4. The results, which are quite similar to the previous analysis, show that the panel cointegration tests with smooth breaks reject the null hypothesis of no cointegration at the 10% significance level, while the panel cointegration tests with sharp breaks and no breaks do not reject the null of no cointegration. In the light of our suggested test results, the PPP hypothesis is valid for OECD countries, which is consistent with theoretical expectations and the results of Omay et al. (2018).

Table 4.4: Results from panel cointegration tests for updated data

	Westerlund and Edgerton (2008)						This study				
	No break		Sharp Breaks				Smooth Breaks				
Country	LM_τ		LM_τ	Break Dates			$LM_\tau(1)$	p-value	$LM_\tau(\hat{m})$	p-value	$\hat{m}$
Austria	-1.316		-0.678	2008Q3	2010Q1	2014Q4	-4.134**	0.045	-4.844***	0.006	1
Belgium	-2.664		-3.557	2008Q3	2010Q1	2014Q4	-4.318**	0.027	-3.805*	0.099	1
Canada	-2.222		-2.009	2004Q3	2008Q3	2014Q4	-2.713	0.627	-3.232	0.798	2
Denmark	-1.041		-1.683	2008Q3	2010Q1	2014Q4	-3.574	0.163	-3.538	0.175	1
Finland	-1.731		-1.272	2008Q3	2010Q1	2014Q4	-3.671	0.133	-3.601	0.154	1
France	-1.838		-2.200	2008Q3	2010Q1	2014Q4	-2.665	0.658	-2.906	0.500	1
Germany	-0.316		-1.574	2008Q3	2010Q1	2014Q4	-3.870*	0.085	-3.695	0.126	1
Greece	-1.296		-1.854	2008Q3	2010Q1	2014Q4	-2.920	0.491	-3.313	0.268	1
Italy	-0.187		-0.222	2008Q3	2010Q1	2014Q4	-2.171	0.918	-2.784	0.580	1
Japan	-1.626		-2.934	2008Q3	2012Q4	2014Q3	-2.739	0.610	-3.333	0.743	2
Netherlands	-2.036		-2.235	2008Q3	2010Q1	2014Q4	-2.301	0.866	-2.293	0.869	1
Norway	-2.746		-2.638	2008Q3	2014Q3	2014Q4	-3.615	0.150	-3.772	0.107	1
Portugal	-1.148		-1.107	2008Q3	2010Q1	2014Q4	-2.304	0.864	-2.306	0.863	1
Spain	-1.361		-2.021	2008Q3	2010Q1	2014Q4	-2.965	0.463	-2.683	0.647	1
Sweden	-2.871		-2.379	2008Q3	2014Q4	2020Q2	-3.293	0.278	-3.284	0.282	1
Switzerland	-2.174		-2.718	2008Q3	2011Q1	2011Q3	-3.053	0.408	-3.479	0.197	1
UK	-3.176		-2.113	2008Q3	2008Q4	2016Q2	-4.214**	0.036	-3.484	0.195	1
Panel Results	Statistic	p-value	Statistic	p-value			Statistic	p-value	Statistic	p-value	
Z_τ	1.560	0.941	0.106	0.542			-1.713**	0.043			
P							47.265*	0.065	47.400*	0.063	
P_m							1.609*	0.054	1.625*	0.052	
# Factors	5		5				5		5		

Notes: See Table 4.2.

4.4. Conclusion

In this chapter, we employ our new panel cointegration tests to examine the validity of PPP hypothesis. In order to compare, we also apply the panel cointegration tests developed by Westerlund and Edgerton (2008), which allow multiple sharp breaks and cross-sectional dependence in DGP. Additionally, we use both the dataset used by Westerlund and Edgerton (2008) and the updated dataset for the sake of accurate comparison. The results from cointegration test of Westerlund and Edgerton (2008) show that the null of cointegration cannot be rejected, and the results imply the PPP hypothesis is invalid. However, the results from our suggested cointegration tests are reversed in favor of cointegration, which is consistent with theoretical expectations and the findings in Drine and Rault (2008) and Omay et al. (2018) for both datasets. This conclusion emphasizes the importance of accounting for structural breaks when conducting cointegration tests.

5. CONCLUSION

This dissertation focuses on testing the cointegration in time series and panel data analysis and suggests new cointegration tests for both contexts. Moreover, this study also re-examining the purchasing power parity hypothesis for OECD countries to demonstrate the practical application of the new tests. The conclusion part of this thesis briefly summarizes the findings of these studies.

In first part of this dissertation, we develop a new cointegration test in the presence of unknown forms of structural changes. The new test is an extension of unit root test proposed by Enders and Lee (2012) with multivariate cointegration context. The important feature of this test is that it can admit serially correlated and heteroskedastic errors, deterministic trends, and smooth structural breaks. The Fourier approximation suggest by Gallant (1981) is used to consider structural breaks. The advantage of this approach is that a small number of Fourier terms mimics well for various types of breaks and it doesn't require a priori information such as the number, location, or form of breaks. The asymptotic distribution of proposed test statistic is derived, and the asymptotic results show that the distribution depends only Fourier frequency. The asymptotic analysis also reveals that the new test is independent of the regressors. Making the testing procedure relatively simple and easy to implement, we present the convenient response surface estimates to obtain finite-sample critical values and corresponding p-values. Monte Carlo simulations are conducted to evaluate the finite-sample size and power of the suggested test. Our simulation results reveal that proposed test perform reasonably well for the different specification of structural breaks, with improved size and power properties.

The second part of this study suggests new cointegration tests with structural breaks for dependent panels. Regarding structural breaks and how to handle cross-sectional dependence, our tests are different from panel cointegration literature. We employ the Fourier approximation suggested by Gallant (1981) to capture multiple structural breaks. It can capture unknown multiple structural changes by using small number of frequencies (Nazlioglu et al., 2022) and it does not require a priori information such as the number, location, or form of breaks. On the other hand, we utilize the PANIC approach of Bai and

Ng (2004) to take into account the cross-sectional dependence and show that our panel cointegration tests do not depend on the factor structure or unknown forms of cross-correlations of the error terms. The asymptotic results of our tests also indicate that the distribution of test statistics only depends on the Fourier frequency and independent of the number of regressors. Finally, Monte Carlo simulations are used to obtain the finite-sample size and power of the suggested tests. The simulation results reveal that proposed tests are good size and power properties for stationary or non-stationary common factors under the different types of structural breaks.

The last part of this dissertation, we examine the validity of PPP hypothesis for 17 OECD countries by applying our new panel cointegration tests. For comparison purpose, we also employ the panel cointegration tests developed by Westerlund and Edgerton (2008), which allow multiple sharp breaks and cross-sectional dependence in DGP. Moreover, we utilize both the dataset used by Westerlund and Edgerton (2008) and the updated dataset for the sake of accurate comparison. The results from cointegration test of Westerlund and Edgerton (2008) show that the null of cointegration cannot be rejected, and the results imply the PPP hypothesis is invalid for 17 OECD countries. However, the results from our suggested cointegration tests are reversed in favor of cointegration, which is consistent with theoretical expectations and the findings in Drine and Rault (2008) and Omay et al. (2018) for both datasets. This conclusion emphasizes the importance of accounting for structural breaks when conducting cointegration tests.

6. REFERENCES

Adda, J., Gonzalo, J. (1996). P-values for non-standard distributions with an application to the DF test. *Economics Letters*, 50:155–160.

Arai, Y., Kurozumi, E. (2007). Testing for the null hypothesis of cointegration with a structural break. *Econometric Reviews*, *26*(6):705-739.

Arize, A. C., Malindretos, J., Ghosh, D. (2015). Purchasing power parity-symmetry and proportionality: Evidence from 116 countries. *International Review of Economics & Finance*, 37:69-85.

Asteriou, D., Hall S. G. (2007). *Applied Econometrics a Modern Approach*, Revised Edition, Palgrave Macmillan, Hampshire.

Bai, J. Carrion-i-Silvestre, J. L. (2009). Structural changes, common stochastic trends, and unit roots in panel data. *Review of Economic Studies*, 76:471–501.

Bai, J., Carrion-i-Silvestre, J. L. (2013). Testing panel cointegration with unobservable dynamic common factors that are correlated with the regressors. *The Econometrics Journal*, 16(2):222-249.

Bai, J., Ng, S. (2002). Determining the number of factors in approximate factor models. *Econometrica*, 70(1):191-221.

Bai, J., Ng, S. (2004). A PANIC attack on unit roots and cointegration. *Econometrica*, 72(4):1127–1177.

Bai, J., Ng, S. (2010). Panel unit root tests with cross-section dependence: a further investigation. *Econometric Theory*, 26(4):1088-1114.

Banerjee, A., Carrion-i-Silvestre, J. L. (2015). Cointegration in panel data with structural breaks and cross-section dependence. *Journal of Applied Econometrics*, 30(1):1-23.

Banerjee, P., Arčabić, V., Lee, H. (2017). Fourier ADL cointegration test to approximate smooth breaks with new evidence from crude oil market. *Economic Modelling*, 67:114-124.

Banerjee, A., Carrion-i-Silvestre, J. L. (2017). Testing for panel cointegration using common correlated effects estimators. *Journal of Time Series Analysis*, 38(4):610-636.

Becker, R., Enders, W., Lee, J. (2006). A stationarity test in the presence of an unknown number of smooth breaks. *Journal of Time Series Analysis*, *27*(3):381-409.

Bahmani-Oskooee, M., Hegerty, S. W. (2009). Purchasing power parity in less-developed and transition economies: A review paper. *Journal of Economic Surveys*, 23(4):617-658.

Britannica (2023). The Editors of Encyclopaedia. "euro". Encyclopedia Britannica, Invalid Date, https://www.britannica.com/topic/euro. Accessed 31 January 2023.

Cassel, G. (1916). The present situation of the foreign exchanges. *The Economic Journal*, 26(103):319-323.

Cheung, Y-W., Lai, K. S. (1995), Lag order and critical values of the augmented Dickey-Fuller test. *Journal of Business & Economic Statistics*, 13:277–280.

Choi, I. (2001). Unit root tests for panel data. *Journal of international money and Finance*, 20(2):249-272.

Cook, S. (2004). Spurious rejection by cointegration tests incorporating structural change in the cointegrating relationship. *Applied Economics Letters*, *11*(14):879-884.

Drine, I., Rault, C. (2008). Purchasing power parity for developing and developed countries. What can we learn from non-stationary panel data models?. *Journal of Economic Surveys*, 22(4):752-773.

Enders, W., Lee, J. (2012). The flexible Fourier form and Dickey–Fuller type unit root tests. *Economics Letters*, 117:196–199.

Fisher, R.A. (1932). *Statistical Methods for Research Workers*, 4th edition. Oliver and Boyd, Edinburgh.

Gallant, R. (1981). On the basis in flexible functional form and an essentially unbiased form: the flexible Fourier form. *Journal of Econometrics*, 15:211–245.

Gallant, A. R., Souza, G. (1991). On the asymptotic normality of Fourier flexible form estimates. *Journal of Econometrics*, *50*(3):329-353.

Gengenbach, C., Palm, F. C., Urbain, J. P. (2006). Cointegration testing in panels with common factors. *Oxford Bulletin of Economics and Statistics*, 68:683-719.

Gengenbach, C., Urbain, J. P., Westerlund, J. (2016). Error correction testing in panels with common stochastic trends. *Journal of Applied Econometrics*, 31(6):982-1004.

Granger, C. W. (1981). Some properties of time series data and their use in econometric model specification. *Journal of Econometrics*, 16(1):121-130.

Granger, C. W., Newbold, P. (1974). Spurious regressions in econometrics. *Journal of Econometrics*, 2(2), 111-120.

Gregory, A. W., Hansen, B. E. (1996). Residual-based tests for cointegration in models with regime shifts. *Journal of Econometrics*, *70*(1):99-126.

Gutierrez, L. (2010). Simple tests for cointegration in panels with structural breaks. *Applied Economics Letters*, 17(2):197-200.

Hanck, C. (2009). A meta analytic approach to testing for panel cointegration. *Communications in Statistics-Simulation and Computation*, 38(5):1051-1070.

Hadri, K., Kurozumi, E., Rao, Y. (2015). Novel panel cointegration tests emending for cross-section dependence with N fixed. *The Econometrics Journal*, 18(3):363-411.

Hansen, B. E. (1997). Approximate asymptotic p values for structural-change tests. *Journal of Business & Economic Statistics*, 15:60–67.

Hatemi-j, A. (2008). Tests for cointegration with two unknown regime shifts with an application to financial market integration. *Empirical economics*, 35(3):497-505.

Hao, K. (1996). Testing for structural change in cointegrated regression models: some comparisons and generalizations. *Econometric Reviews*, 15(4):401-429.

Harris, D., Leybourne, S., McCabe, B. (2005). Panel stationarity tests for purchasing power parity with cross-sectional dependence. *Journal of Business & Economic Statistics*, 23(4):395-409.

Hegwood, N. D., Papell, D. H. (1998). Quasi purchasing power parity. *International Journal of Finance & Economics*, 3(4):279-289.

Inoue, A. (1999). Tests of cointegrating rank with a trend-break. *Journal of Econometrics, 90*(2):215-237.

Karabiyik, H., Palm, F.C., Urbain, J.P. (2019). Econometric analysis of panel data models with multifactor error structures. *Annual Review of Economics*, 11:495-522.

Kao, C. (1999). Spurious regression and residual-based tests for cointegration in panel data. *Journal of Econometrics*, 90(1):1-44.

Larsson, R., Lyhagen, J., Löthgren, M. (2001). Likelihood-based cointegration tests in heterogeneous panels. *The Econometrics Journal*, 4(1):109-142.

Leybourne, S. J., Newbold, P. (2003). Spurious rejections by cointegration tests induced by structural breaks. *Applied Economics, 35*(9):1117-1121.

Lütkepohl, H., Saikkonen, P., Trenkler, C. (2004). Testing for the cointegrating rank of a VAR process with level shift at unknown time. *Econometrica, 72*(2):647-662.

MacKinnon, J. G. (1996). Numerical distribution functions for unit root and cointegration tests. *Journal of Applied Econometrics,* 11:601–618.

MacKinnon, J. G. (2002). Computing numerical distribution functions in econometrics. In: High-performance computing systems and applications. Boston: Springer, 455–471.

Maddala, G. S., Kim, I. M. (1998). *Unit roots, cointegration, and structural change*. Cambridge University Press, Cambridge.

Maddala, G. S., Wu, S. (1999). A comparative study of unit root tests with panel data and a new simple test. *Oxford Bulletin of Economics and Statistics*, 61(Special Issue):631-652.

Maki, D. (2012). Tests for cointegration allowing for an unknown number of breaks. *Economic Modelling*, 29(5):2011-2015.

Munir, Q., Kok, S. C., Lean, H. H., Teplova, T. (2018). Purchasing power parity in ASEAN-5 countries: Revisit with cross-sectional dependence and structural breaks. *Macroeconomics and Finance in Emerging Market Economies*, 11(3):233-249.

Murray, C. J., Papell, D. H. (2005). The purchasing power parity puzzle is worse than you think. *Empirical Economics*, 30:783-790.

Narayan, P. K. (2005). New evidence on purchasing power parity from 17 OECD countries. *Applied Economics*, 37(9):1063-1071.

Narayan, P. K. (2006). Are bilateral real exchange rates stationary? Evidence from Lagrange multiplier unit root tests for India. *Applied Economics*, 38(1):63-70.

Narayan, P. K. (2007). Are nominal exchange rates and price levels co-integrated? New evidence from threshold autoregressive and momentum-threshold autoregressive models. *Economic Record*, 83(260):74-85.

Narayan, P. K., Narayan, S., Prasad, A. (2009). Evidence on PPP from a cointegration test with multiple structural breaks. *Applied Economics Letters*, 16(1):5-8.

Nazlioglu, S., Lee, J., Tieslau, M., Karul, C., You, Y. (2022). Smooth structural changes and common factors in nonstationary panel data: An analysis of healthcare expenditures. *Econometric Reviews*, doi.org/10.1080/07474938.2022.2156740.

O'Connell, P. G. (1998). The overvaluation of purchasing power parity. *Journal of İnternational Economics*, 44(1):1-19.

Olayeni, R. O., Tiwari, A. K., Wohar, M. E. (2021). Fractional frequency flexible Fourier form (FFFFF) for panel cointegration test. *Applied Economics Letters*, 28(6), 482-486.

Omay, T., Emirmahmutoglu, F., Hasanov, M. (2018). Structural break, nonlinearity and asymmetry: A re-examination of PPP proposition. *Applied Economics*, 50(12):1289-1308.

Otero, J., Smith, J. (2012). Response surface models for the Leybourne unit root tests and lag order dependence. *Computational Statistics*, 27:473–486.

Papell, D. H. (2002). The great appreciation, the great depreciation, and the purchasing power parity hypothesis. *Journal of International Economics*, 57(1):51-82.

Pesaran, M. H. (2006). Estimation and inference in large heterogeneous panels with a multifactor error structure. *Econometrica*, 74(4):967-1012.

Pedroni, P. (1999). Critical values for cointegration tests in heterogeneous panels with multiple regressors. *Oxford Bulletin of Economics and Statistics*, 61:653-670.

Pedroni, P. (2004). Panel cointegration: asymptotic and finite sample properties of pooled time series tests with an application to the PPP hypothesis. *Econometric Theory*, 20:597-625.

Perron, P. (1989), The great crash, the oil price shock, and the unit root hypothesis. *Econometrica*, 57:1361-1401.

Phillips, P. C. (1986). Understanding spurious regressions in econometrics. *Journal of Econometrics*, 33(3):311-340.

Rogoff, K. (1996). The purchasing power parity puzzle. *Journal of Economic literature*, 34(2):647-668.

Salehizadeh, M., Taylor, R. (1999). A test of purchasing power parity for emerging economies. *Journal of International Financial Markets, Institutions and Money*, 9(2):183-193.

Sarno, L., Taylor, M. P. (2002). Purchasing power parity and the real exchange rate. *International Monetary Fund Staff Papers*, 49(1):65-105.

Schmidt, P., Phillips, P. C. (1992). LM tests for a unit root in the presence of deterministic trends. *Oxford Bulletin of Economics and Statistics*, 54:257–287.

Snaith, S. (2012). The PPP debate: multiple breaks and cross-sectional dependence. *Economics Letters*, 115(3):342-344.

Stock, J. H., Watson, M. W. (1996). Evidence on structural instability in macroeconomic time series relations. *Journal of Business & Economic Statistics*, *14*(1):11-30.

Taylor, M. P. (1988). An empirical examination of long-run purchasing power parity using cointegration techniques. *Applied Economics*, 20(10):1369-1381.

Taylor, A. M. (2002). A century of purchasing-power parity. *Review of Economics and Statistics*, 84(1):139-150.

Taylor, M. P., McMahon, P. C. (1988). Long-run purchasing power parity in the 1920s. *European Economic Review*, 32(1):179-197.

Tsong, C. C., Lee, C. F., Tsai, L. J., Hu, T. C. (2016). The Fourier approximation and testing for the null of cointegration. *Empirical Economics*, *51*(3):1085-1113.

Westerlund, J. (2006a). Testing for panel cointegration with a level break. *Economics Letters*, 91(1):27-33.

Westerlund, J. (2006b). Testing for panel cointegration with multiple structural breaks. *Oxford Bulletin of Economics and Statistics*, 68(1):101-132.

Westerlund, J. (2007). Testing for error correction in panel data. *Oxford Bulletin of Economics and Statistics*, 69(6):709-748.

Westerlund, J. (2008). Panel cointegration tests of the Fisher Hypothesis. *Journal of Applied Econometrics*, 23:193–233.

Westerlund, J. (2012). Testing for unit roots in panel time-series models with multiple level breaks. *The Manchester School*, 80(6):671–699.

Westerlund, J., Edgerton, D. L. (2007a). A panel bootstrap cointegration test. *Economics Letters*, 97(3):185-190.

Westerlund, J., Edgerton, D. L. (2007b). New improved tests for cointegration with structural breaks. *Journal of time series Analysis*, *28*(2):188-224.

Westerlund, J., Edgerton, D. L. (2008). A simple test for cointegration in dependent panels with structural breaks. *Oxford Bulletin of Economics and Statistics*, 70(5):665-704.

De Villiers, D., Phiri, A. (2022). Towards resolving the purchasing power parity (PPP)'Puzzle'in newly industrialized countries (NIC's). *The Journal of International Trade & Economic Development*, 31(2):161-180.

Yazgan, M. E. (2003). The purchasing power parity hypothesis for a high inflation country: a re-examination of the case of Turkey. *Applied Economics Letters*, 10(3), 143-147.

7. APPENDICES

7.1. Appendix A

Table 7.1. Computed critical values using the response surface estimates for cumulative Fourier frequencies

		m=1			*m=2*			*m=3*		
p	T	0.01	0.05	0.10	0.01	0.05	0.10	0.01	0.05	0.10
0	50	-4.85	-4.21	-3.89	-5.75	-5.07	-4.74	-6.57	-5.84	-5.48
	100	-4.69	-4.11	-3.83	-5.50	-4.91	-4.62	-6.19	-5.59	-5.29
	200	-4.62	-4.07	-3.80	-5.39	-4.84	-4.56	-6.03	-5.49	-5.21
	500	-4.57	-4.04	-3.78	-5.32	-4.80	-4.53	-5.94	-5.43	-5.16
	2500	-4.54	-4.03	-3.76	-5.29	-4.77	-4.51	-5.89	-5.39	-5.14
1	50	-4.85	-4.21	-3.89	-5.76	-5.07	-4.73	-6.52	-5.82	-5.48
	100	-4.69	-4.12	-3.82	-5.50	-4.91	-4.61	-6.14	-5.58	-5.29
	200	-4.61	-4.08	-3.79	-5.39	-4.84	-4.56	-5.99	-5.48	-5.20
	500	-4.57	-4.05	-3.78	-5.32	-4.80	-4.53	-5.91	-5.42	-5.16
	2500	-4.55	-4.04	-3.77	-5.29	-4.78	-4.52	-5.88	-5.39	-5.13
2	50	-4.85	-4.21	-3.89	-5.76	-5.07	-4.73	-6.53	-5.83	-5.48
	100	-4.69	-4.12	-3.82	-5.50	-4.91	-4.62	-6.15	-5.58	-5.29
	200	-4.61	-4.08	-3.79	-5.39	-4.84	-4.56	-5.99	-5.48	-5.20
	500	-4.57	-4.05	-3.78	-5.32	-4.80	-4.53	-5.91	-5.42	-5.16
	2500	-4.55	-4.03	-3.77	-5.29	-4.78	-4.52	-5.88	-5.39	-5.13
3	50	-4.85	-4.21	-3.89	-5.76	-5.07	-4.73	-6.55	-5.83	-5.48
	100	-4.69	-4.12	-3.82	-5.50	-4.91	-4.62	-6.16	-5.58	-5.29
	200	-4.61	-4.07	-3.79	-5.39	-4.84	-4.56	-6.00	-5.48	-5.20
	500	-4.57	-4.05	-3.78	-5.32	-4.80	-4.53	-5.92	-5.42	-5.16
	2500	-4.55	-4.03	-3.77	-5.29	-4.78	-4.52	-5.88	-5.39	-5.13
4	50	-4.85	-4.21	-3.89	-5.76	-5.07	-4.73	-6.56	-5.83	-5.48
	100	-4.69	-4.12	-3.82	-5.50	-4.91	-4.62	-6.16	-5.58	-5.29
	200	-4.61	-4.07	-3.79	-5.39	-4.84	-4.56	-6.00	-5.48	-5.20
	500	-4.57	-4.05	-3.78	-5.32	-4.80	-4.53	-5.92	-5.42	-5.16
	2500	-4.55	-4.03	-3.77	-5.29	-4.78	-4.52	-5.88	-5.39	-5.13

Notes: T is the sample size, m is the cumulative Fourier frequencies, and p is the lag order.

Table 7.2. Computed critical values using the response surface estimates for single Fourier frequency

		k=1			*k=2*			*k=3*			*k=4*			*k=5*		
p	T	0.01	0.05	0.10	0.01	0.05	0.10	0.01	0.05	0.10	0.01	0.05	0.10	0.01	0.05	0.10
0	50	-4.85	-4.20	-3.89	-4.33	-3.60	-3.23	-4.04	-3.30	-2.94	-3.86	-3.16	-2.82	-3.76	-3.09	-2.77
	100	-4.69	-4.11	-3.82	-4.24	-3.57	-3.23	-3.98	-3.30	-2.96	-3.83	-3.18	-2.86	-3.75	-3.11	-2.80
	200	-4.62	-4.07	-3.79	-4.19	-3.56	-3.22	-3.95	-3.30	-2.97	-3.82	-3.19	-2.87	-3.74	-3.13	-2.82
	500	-4.58	-4.05	-3.78	-4.15	-3.55	-3.22	-3.93	-3.30	-2.98	-3.81	-3.19	-2.88	-3.74	-3.13	-2.83
	2500	-4.55	-4.04	-3.77	-4.12	-3.54	-3.22	-3.93	-3.31	-2.99	-3.81	-3.19	-2.88	-3.74	-3.14	-2.84
1	50	-4.88	-4.20	-3.90	-4.32	-3.61	-3.24	-4.03	-3.32	-2.94	-3.88	-3.16	-2.83	-3.77	-3.09	-2.77
	100	-4.72	-4.11	-3.84	-4.23	-3.58	-3.24	-3.98	-3.32	-2.97	-3.85	-3.18	-2.86	-3.76	-3.12	-2.81
	200	-4.64	-4.07	-3.80	-4.18	-3.56	-3.23	-3.95	-3.31	-2.98	-3.83	-3.19	-2.87	-3.74	-3.13	-2.82
	500	-4.59	-4.04	-3.78	-4.16	-3.55	-3.22	-3.94	-3.31	-2.98	-3.82	-3.19	-2.88	-3.72	-3.13	-2.83
	2500	-4.56	-4.03	-3.77	-4.15	-3.54	-3.22	-3.92	-3.30	-2.98	-3.82	-3.19	-2.88	-3.71	-3.13	-2.83
2	50	-4.87	-4.20	-3.90	-4.32	-3.61	-3.24	-4.03	-3.31	-2.94	-3.87	-3.16	-2.83	-3.77	-3.09	-2.77
	100	-4.72	-4.11	-3.83	-4.23	-3.58	-3.24	-3.98	-3.32	-2.97	-3.84	-3.18	-2.86	-3.76	-3.12	-2.81
	200	-4.64	-4.07	-3.80	-4.18	-3.56	-3.23	-3.95	-3.31	-2.98	-3.83	-3.19	-2.87	-3.74	-3.13	-2.82
	500	-4.59	-4.04	-3.78	-4.16	-3.55	-3.22	-3.93	-3.31	-2.98	-3.82	-3.19	-2.88	-3.72	-3.13	-2.83
	2500	-4.56	-4.03	-3.77	-4.15	-3.54	-3.22	-3.92	-3.30	-2.98	-3.82	-3.19	-2.88	-3.71	-3.13	-2.83
3	50	-4.86	-4.20	-3.89	-4.32	-3.61	-3.24	-4.03	-3.31	-2.94	-3.86	-3.16	-2.82	-3.77	-3.09	-2.77
	100	-4.71	-4.11	-3.83	-4.23	-3.58	-3.23	-3.98	-3.31	-2.96	-3.84	-3.18	-2.86	-3.76	-3.12	-2.81
	200	-4.63	-4.07	-3.80	-4.18	-3.56	-3.23	-3.95	-3.31	-2.98	-3.83	-3.19	-2.87	-3.74	-3.13	-2.82
	500	-4.58	-4.04	-3.78	-4.16	-3.55	-3.22	-3.93	-3.31	-2.98	-3.82	-3.19	-2.88	-3.72	-3.13	-2.83
	2500	-4.56	-4.03	-3.77	-4.15	-3.54	-3.22	-3.92	-3.30	-2.98	-3.82	-3.19	-2.88	-3.71	-3.13	-2.83
4	50	-4.86	-4.20	-3.89	-4.33	-3.61	-3.24	-4.03	-3.31	-2.94	-3.86	-3.16	-2.82	-3.77	-3.09	-2.77
	100	-4.70	-4.11	-3.83	-4.23	-3.58	-3.23	-3.98	-3.31	-2.96	-3.84	-3.18	-2.86	-3.76	-3.12	-2.81
	200	-4.63	-4.07	-3.80	-4.18	-3.56	-3.23	-3.95	-3.31	-2.97	-3.83	-3.19	-2.87	-3.74	-3.13	-2.82
	500	-4.58	-4.04	-3.78	-4.16	-3.55	-3.22	-3.93	-3.31	-2.98	-3.82	-3.19	-2.88	-3.72	-3.13	-2.83
	2500	-4.56	-4.03	-3.77	-4.15	-3.54	-3.22	-3.92	-3.30	-2.98	-3.82	-3.19	-2.88	-3.71	-3.13	-2.83

Notes: T is the sample size, k is the single Fourier frequency, and p is the lag order.

Table 7.3. Size properties of the LM-based cointegration test statistics with $h = 2$

B. Type	θ	φ	T	$\tau_{FLM}(1)$	$\tau_{FLM}(2)$	$\tau_{FLM}(3)$	$\tau_{FLM}(\hat{m})$	t_C	ϕ_C	t_N	ϕ_N
No break	0	0	100	0.046	0.051	0.060	0.056	0.062	0.077	0.060	0.073
			200	0.050	0.054	0.056	0.055	0.048	0.056	0.049	0.055
			300	0.049	0.052	0.054	0.053	0.053	0.057	0.052	0.059
	0.4	0	100	0.043	0.051	0.053	0.057	0.059	0.022	0.054	0.019
			200	0.049	0.050	0.055	0.061	0.058	0.015	0.058	0.014
			300	0.049	0.045	0.055	0.062	0.048	0.014	0.050	0.014
	-0.4	0	100	0.044	0.047	0.052	0.044	0.049	0.214	0.051	0.217
			200	0.050	0.053	0.058	0.050	0.053	0.224	0.053	0.226
			300	0.047	0.046	0.053	0.047	0.050	0.233	0.049	0.238
	0	0.4	100	0.049	0.054	0.064	0.068	0.061	0.013	0.061	0.012
			200	0.051	0.054	0.061	0.073	0.062	0.008	0.060	0.008
			300	0.043	0.046	0.058	0.063	0.049	0.005	0.050	0.006
	0	-0.4	100	0.041	0.049	0.049	0.041	0.059	0.171	0.061	0.176
			200	0.048	0.052	0.056	0.048	0.046	0.149	0.046	0.152
			300	0.050	0.051	0.049	0.050	0.050	0.156	0.047	0.160
Sharp	0	0	100	0.042	0.049	0.058	0.051	0.059	0.074	0.063	0.078
			200	0.046	0.048	0.051	0.052	0.052	0.060	0.052	0.056
			300	0.040	0.048	0.055	0.047	0.052	0.057	0.051	0.058
	0.4	0	100	0.047	0.052	0.059	0.066	0.059	0.024	0.052	0.026
			200	0.046	0.049	0.053	0.058	0.053	0.014	0.051	0.015
			300	0.046	0.051	0.059	0.062	0.048	0.015	0.046	0.015
	-0.4	0	100	0.042	0.046	0.047	0.042	0.060	0.226	0.053	0.208
			200	0.057	0.057	0.057	0.057	0.055	0.230	0.051	0.220
			300	0.050	0.046	0.052	0.050	0.048	0.236	0.050	0.213
	0	0.4	100	0.043	0.051	0.063	0.064	0.066	0.016	0.064	0.015
			200	0.049	0.048	0.062	0.071	0.055	0.007	0.053	0.009
			300	0.047	0.054	0.061	0.065	0.051	0.006	0.054	0.007
	0	-0.4	100	0.042	0.041	0.051	0.042	0.060	0.176	0.056	0.162
			200	0.046	0.053	0.053	0.046	0.049	0.150	0.048	0.150
			300	0.042	0.051	0.052	0.042	0.053	0.166	0.053	0.161
Fourier	0	0	100	0.045	0.050	0.051	0.048	0.026	0.016	0.027	0.015
			200	0.048	0.048	0.053	0.055	0.032	0.025	0.032	0.025
			300	0.049	0.052	0.059	0.057	0.034	0.034	0.034	0.035
	0.4	0	100	0.043	0.046	0.059	0.056	0.038	0.006	0.037	0.006
			200	0.048	0.042	0.050	0.059	0.037	0.010	0.037	0.010
			300	0.054	0.060	0.060	0.067	0.041	0.009	0.040	0.009
	-0.4	0	100	0.045	0.052	0.051	0.045	0.020	0.012	0.020	0.012
			200	0.053	0.050	0.056	0.053	0.019	0.045	0.021	0.045
			300	0.049	0.053	0.051	0.049	0.019	0.073	0.019	0.072
	0	0.4	100	0.050	0.057	0.063	0.064	0.046	0.008	0.045	0.009
			200	0.046	0.054	0.058	0.064	0.049	0.006	0.050	0.006
			300	0.047	0.054	0.061	0.067	0.041	0.003	0.043	0.003
	0	-0.4	100	0.045	0.048	0.053	0.045	0.026	0.019	0.025	0.018
			200	0.043	0.049	0.057	0.043	0.023	0.041	0.022	0.043

			300	0.048	0.048	0.055	0.048	0.028	0.066	0.028	0.065
LSTAR	0	0	100	0.046	0.044	0.054	0.047	0.054	0.129	0.055	0.132
			200	0.051	0.048	0.050	0.051	0.051	0.159	0.049	0.160
			300	0.046	0.050	0.055	0.046	0.046	0.177	0.047	0.178
	0.4	0	100	0.047	0.051	0.063	0.055	0.058	0.069	0.059	0.068
			200	0.047	0.049	0.055	0.051	0.054	0.080	0.053	0.080
			300	0.046	0.051	0.053	0.048	0.050	0.088	0.047	0.086
	-0.4	0	100	0.053	0.057	0.059	0.053	0.051	0.256	0.055	0.263
			200	0.081	0.090	0.096	0.081	0.057	0.333	0.058	0.341
			300	0.079	0.082	0.083	0.079	0.053	0.371	0.056	0.382
	0	0.4	100	0.048	0.049	0.060	0.062	0.058	0.040	0.057	0.038
			200	0.049	0.051	0.059	0.057	0.046	0.043	0.047	0.042
			300	0.052	0.058	0.058	0.059	0.048	0.047	0.047	0.049
	0	-0.4	100	0.047	0.048	0.046	0.047	0.052	0.203	0.056	0.205
			200	0.056	0.056	0.063	0.056	0.050	0.266	0.049	0.271
			300	0.056	0.062	0.063	0.056	0.048	0.309	0.052	0.317
ESTAR	0	0	100	0.047	0.047	0.055	0.049	0.053	0.116	0.054	0.115
			200	0.044	0.050	0.057	0.045	0.051	0.144	0.051	0.144
			300	0.049	0.050	0.050	0.049	0.054	0.151	0.054	0.146
	0.4	0	100	0.041	0.049	0.060	0.050	0.056	0.057	0.058	0.055
			200	0.050	0.047	0.057	0.054	0.052	0.071	0.056	0.075
			300	0.052	0.050	0.058	0.053	0.055	0.070	0.053	0.068
	-0.4	0	100	0.053	0.054	0.060	0.053	0.065	0.289	0.063	0.292
			200	0.078	0.080	0.082	0.078	0.058	0.359	0.057	0.364
			300	0.060	0.068	0.071	0.060	0.053	0.372	0.053	0.378
	0	0.4	100	0.051	0.059	0.062	0.062	0.062	0.033	0.063	0.032
			200	0.044	0.049	0.059	0.052	0.057	0.044	0.056	0.046
			300	0.047	0.048	0.060	0.057	0.058	0.043	0.058	0.042
	0	-0.4	100	0.043	0.049	0.057	0.043	0.060	0.228	0.063	0.219
			200	0.049	0.051	0.053	0.049	0.056	0.265	0.057	0.266
			300	0.042	0.046	0.051	0.042	0.058	0.279	0.059	0.282

Notes: See Table 2.5.

Table 7.4. Power properties of the LM-based cointegration test statistics with $h = 2$

B. Type	θ	φ	T	$\tau_{FLM}(1)$	$\tau_{FLM}(2)$	$\tau_{FLM}(3)$	$\tau_{FLM}(\hat{m})$	t_C	ϕ_C	t_N	ϕ_N
No break	0	0	100	0.160	0.095	0.081	0.160	0.234	0.417	0.297	0.450
			200	0.605	0.386	0.263	0.605	0.686	0.950	0.686	0.949
			300	0.884	0.706	0.525	0.884	0.861	0.856	0.919	0.922
	0.4	0	100	0.146	0.086	0.078	0.148	0.236	0.564	0.234	0.502
			200	0.568	0.342	0.228	0.568	0.707	0.466	0.767	0.461
			300	0.884	0.691	0.502	0.884	0.921	0.954	0.938	0.961
	-0.4	0	100	0.149	0.104	0.088	0.149	0.217	0.802	0.235	0.818
			200	0.607	0.428	0.299	0.607	0.572	0.760	0.646	0.810
			300	0.857	0.713	0.573	0.857	0.730	0.997	0.797	0.997
	0	0.4	100	0.128	0.093	0.079	0.134	0.290	0.052	0.273	0.052
			200	0.547	0.329	0.215	0.548	0.763	0.717	0.802	0.725
			300	0.868	0.654	0.472	0.868	0.946	0.993	0.949	0.992
	0	-0.4	100	0.140	0.093	0.077	0.140	0.226	0.311	0.254	0.357
			200	0.561	0.372	0.250	0.561	0.642	0.991	0.685	0.994
			300	0.837	0.693	0.527	0.837	0.790	0.935	0.829	0.954
Sharp	0	0	100	0.127	0.085	0.079	0.127	0.220	0.402	0.201	0.305
			200	0.479	0.316	0.227	0.479	0.688	0.949	0.487	0.875
			300	0.793	0.627	0.470	0.793	0.852	0.847	0.792	0.792
	0.4	0	100	0.129	0.094	0.088	0.133	0.239	0.551	0.189	0.420
			200	0.515	0.320	0.233	0.515	0.698	0.465	0.649	0.337
			300	0.836	0.640	0.488	0.836	0.919	0.949	0.860	0.913
	-0.4	0	100	0.093	0.070	0.066	0.093	0.204	0.805	0.110	0.655
			200	0.383	0.280	0.216	0.383	0.557	0.758	0.349	0.538
			300	0.642	0.537	0.430	0.642	0.741	0.996	0.533	0.988
	0	0.4	100	0.131	0.092	0.072	0.139	0.296	0.054	0.233	0.047
			200	0.502	0.303	0.218	0.502	0.760	0.715	0.716	0.624
			300	0.821	0.619	0.456	0.821	0.952	0.993	0.916	0.979
	0	-0.4	100	0.102	0.076	0.070	0.102	0.230	0.298	0.142	0.177
			200	0.404	0.288	0.201	0.404	0.656	0.993	0.445	0.979
			300	0.673	0.542	0.431	0.673	0.796	0.927	0.612	0.856
Fourier	0	0	100	0.160	0.094	0.086	0.161	0.005	0.000	0.011	0.000
			200	0.608	0.381	0.269	0.608	0.007	0.010	0.005	0.009
			300	0.895	0.712	0.529	0.895	0.005	0.000	0.012	0.000
	0.4	0	100	0.138	0.095	0.079	0.141	0.016	0.012	0.016	0.007
			200	0.583	0.359	0.237	0.583	0.025	0.000	0.037	0.000
			300	0.889	0.697	0.518	0.889	0.074	0.025	0.080	0.026
	-0.4	0	100	0.147	0.097	0.078	0.147	0.009	0.003	0.008	0.003
			200	0.612	0.425	0.308	0.612	0.002	0.000	0.004	0.000
			300	0.843	0.724	0.573	0.843	0.001	0.146	0.002	0.127
	0	0.4	100	0.145	0.094	0.082	0.150	0.048	0.000	0.043	0.001
			200	0.563	0.345	0.234	0.565	0.087	0.015	0.109	0.016
			300	0.861	0.657	0.475	0.861	0.199	0.367	0.194	0.350
	0	-0.4	100	0.139	0.093	0.078	0.139	0.010	0.000	0.009	0.000
			200	0.581	0.388	0.255	0.581	0.003	0.115	0.004	0.108

			300	0.840	0.692	0.523	0.840	0.002	0.000	0.002	0.000
LSTAR	0	0	100	0.152	0.100	0.081	0.152	0.137	0.377	0.192	0.418
			200	0.638	0.421	0.284	0.638	0.455	0.891	0.458	0.903
			300	0.898	0.736	0.566	0.898	0.589	0.761	0.707	0.865
	0.4	0	100	0.150	0.098	0.087	0.151	0.180	0.626	0.185	0.578
			200	0.614	0.401	0.265	0.614	0.529	0.607	0.608	0.632
			300	0.896	0.725	0.540	0.896	0.779	0.956	0.814	0.964
	-0.4	0	100	0.175	0.113	0.092	0.175	0.106	0.632	0.122	0.642
			200	0.691	0.515	0.393	0.691	0.290	0.534	0.347	0.594
			300	0.872	0.785	0.663	0.872	0.406	0.974	0.475	0.982
	0	0.4	100	0.137	0.091	0.081	0.138	0.229	0.111	0.214	0.112
			200	0.584	0.371	0.249	0.584	0.623	0.818	0.678	0.842
			300	0.875	0.695	0.513	0.875	0.847	0.991	0.868	0.994
	0	-0.4	100	0.144	0.103	0.085	0.144	0.137	0.227	0.144	0.263
			200	0.617	0.430	0.308	0.617	0.368	0.948	0.404	0.960
			300	0.869	0.746	0.591	0.869	0.488	0.802	0.540	0.863
ESTAR	0	0	100	0.157	0.104	0.092	0.157	0.215	0.500	0.281	0.535
			200	0.614	0.403	0.269	0.614	0.621	0.961	0.629	0.965
			300	0.884	0.727	0.560	0.884	0.771	0.875	0.863	0.936
	0.4	0	100	0.153	0.094	0.083	0.154	0.224	0.673	0.230	0.626
			200	0.607	0.387	0.260	0.607	0.662	0.723	0.717	0.727
			300	0.899	0.711	0.532	0.899	0.887	0.979	0.910	0.986
	-0.4	0	100	0.167	0.099	0.084	0.167	0.212	0.822	0.231	0.830
			200	0.645	0.483	0.359	0.645	0.495	0.746	0.579	0.790
			300	0.850	0.743	0.632	0.850	0.640	0.993	0.718	0.994
	0	0.4	100	0.138	0.091	0.083	0.140	0.292	0.122	0.280	0.124
			200	0.572	0.360	0.241	0.572	0.741	0.885	0.792	0.899
			300	0.881	0.687	0.494	0.881	0.922	0.999	0.929	0.999
	0	-0.4	100	0.136	0.090	0.072	0.136	0.225	0.340	0.238	0.387
			200	0.593	0.413	0.291	0.593	0.574	0.992	0.618	0.994
			300	0.846	0.712	0.558	0.846	0.700	0.913	0.749	0.944

Notes: See Table 2.5.

Table 7.5. Size properties of the LM-based cointegration test statistics with $h = 3$

B. Type	θ	φ	T	$\tau_{FLM}(1)$	$\tau_{FLM}(2)$	$\tau_{FLM}(3)$	$\tau_{FLM}(\hat{m})$	t_C	ϕ_C	t_N	ϕ_N
No break	0	0	100	0.039	0.050	0.057	0.045	0.058	0.071	0.057	0.073
			200	0.052	0.051	0.054	0.055	0.054	0.065	0.054	0.064
			300	0.053	0.056	0.056	0.057	0.049	0.056	0.050	0.057
	0.4	0	100	0.049	0.050	0.063	0.065	0.064	0.024	0.063	0.024
			200	0.044	0.047	0.056	0.058	0.054	0.015	0.054	0.015
			300	0.040	0.048	0.058	0.051	0.049	0.014	0.048	0.012
	-0.4	0	100	0.044	0.047	0.050	0.044	0.060	0.228	0.060	0.227
			200	0.053	0.054	0.054	0.053	0.059	0.232	0.061	0.235
			300	0.049	0.051	0.056	0.049	0.048	0.213	0.046	0.217
	0	0.4	100	0.044	0.050	0.063	0.062	0.058	0.010	0.058	0.009
			200	0.047	0.053	0.063	0.070	0.053	0.007	0.052	0.007
			300	0.053	0.053	0.054	0.070	0.052	0.007	0.055	0.006
	0	-0.4	100	0.040	0.045	0.046	0.040	0.055	0.161	0.051	0.161
			200	0.051	0.049	0.052	0.051	0.049	0.153	0.049	0.150
			300	0.049	0.051	0.048	0.049	0.050	0.161	0.050	0.162
Sharp	0	0	100	0.043	0.047	0.060	0.051	0.058	0.066	0.057	0.071
			200	0.047	0.045	0.055	0.055	0.058	0.067	0.057	0.065
			300	0.047	0.049	0.054	0.052	0.052	0.058	0.052	0.056
	0.4	0	100	0.046	0.050	0.056	0.061	0.062	0.026	0.058	0.024
			200	0.048	0.046	0.049	0.058	0.053	0.014	0.050	0.015
			300	0.046	0.055	0.060	0.062	0.050	0.013	0.048	0.016
	-0.4	0	100	0.042	0.043	0.051	0.042	0.058	0.221	0.052	0.196
			200	0.049	0.052	0.061	0.049	0.054	0.219	0.048	0.205
			300	0.049	0.056	0.059	0.050	0.052	0.231	0.051	0.220
	0	0.4	100	0.049	0.050	0.065	0.067	0.065	0.011	0.057	0.013
			200	0.049	0.052	0.056	0.066	0.046	0.005	0.048	0.004
			300	0.046	0.054	0.061	0.068	0.055	0.006	0.052	0.006
	0	-0.4	100	0.041	0.045	0.047	0.041	0.058	0.162	0.055	0.156
			200	0.047	0.048	0.052	0.047	0.046	0.147	0.046	0.147
			300	0.047	0.048	0.052	0.047	0.048	0.157	0.051	0.152
Fourier	0	0	100	0.045	0.051	0.055	0.050	0.032	0.015	0.033	0.015
			200	0.040	0.046	0.057	0.045	0.031	0.025	0.032	0.027
			300	0.047	0.054	0.055	0.052	0.039	0.036	0.038	0.037
	0.4	0	100	0.051	0.043	0.059	0.059	0.046	0.010	0.044	0.009
			200	0.047	0.049	0.052	0.059	0.040	0.010	0.040	0.010
			300	0.049	0.052	0.054	0.059	0.042	0.011	0.040	0.011
	-0.4	0	100	0.045	0.053	0.056	0.045	0.024	0.016	0.024	0.015
			200	0.043	0.055	0.056	0.042	0.019	0.044	0.019	0.046
			300	0.054	0.051	0.049	0.054	0.024	0.072	0.023	0.074
	0	0.4	100	0.051	0.053	0.071	0.071	0.051	0.007	0.052	0.007
			200	0.046	0.046	0.055	0.063	0.044	0.003	0.043	0.002
			300	0.047	0.046	0.053	0.063	0.044	0.005	0.043	0.005
	0	-0.4	100	0.036	0.047	0.047	0.036	0.023	0.017	0.024	0.016
			200	0.043	0.045	0.056	0.044	0.023	0.048	0.024	0.049

			300	0.047	0.049	0.050	0.047	0.027	0.059	0.025	0.060
LSTAR	0	0	100	0.040	0.046	0.048	0.042	0.053	0.121	0.052	0.123
			200	0.049	0.053	0.059	0.050	0.049	0.149	0.049	0.151
			300	0.048	0.048	0.056	0.048	0.043	0.166	0.045	0.166
	0.4	0	100	0.046	0.054	0.055	0.051	0.063	0.065	0.058	0.068
			200	0.043	0.052	0.056	0.047	0.056	0.080	0.055	0.080
			300	0.051	0.050	0.056	0.053	0.050	0.094	0.052	0.095
	-0.4	0	100	0.056	0.055	0.059	0.056	0.053	0.249	0.054	0.260
			200	0.086	0.089	0.093	0.086	0.053	0.323	0.053	0.322
			300	0.074	0.087	0.087	0.074	0.048	0.372	0.050	0.375
	0	0.4	100	0.047	0.047	0.056	0.055	0.056	0.038	0.055	0.038
			200	0.049	0.051	0.056	0.056	0.058	0.044	0.057	0.045
			300	0.049	0.057	0.063	0.056	0.049	0.054	0.047	0.056
	0	-0.4	100	0.043	0.044	0.049	0.043	0.053	0.205	0.053	0.213
			200	0.053	0.059	0.056	0.053	0.049	0.260	0.053	0.273
			300	0.057	0.059	0.066	0.057	0.047	0.295	0.049	0.297
ESTAR	0	0	100	0.041	0.046	0.058	0.044	0.059	0.115	0.058	0.116
			200	0.045	0.049	0.053	0.046	0.051	0.150	0.050	0.147
			300	0.047	0.047	0.050	0.048	0.052	0.148	0.051	0.151
	0.4	0	100	0.046	0.051	0.057	0.054	0.061	0.059	0.059	0.059
			200	0.041	0.050	0.049	0.044	0.055	0.074	0.053	0.071
			300	0.051	0.048	0.050	0.055	0.048	0.068	0.048	0.069
	-0.4	0	100	0.055	0.060	0.056	0.055	0.058	0.276	0.061	0.284
			200	0.071	0.080	0.077	0.071	0.061	0.345	0.061	0.343
			300	0.065	0.069	0.074	0.065	0.057	0.370	0.055	0.367
	0	0.4	100	0.047	0.058	0.069	0.061	0.061	0.032	0.062	0.033
			200	0.051	0.057	0.063	0.063	0.054	0.041	0.055	0.043
			300	0.051	0.053	0.055	0.064	0.049	0.037	0.050	0.035
	0	-0.4	100	0.042	0.045	0.053	0.041	0.052	0.215	0.055	0.218
			200	0.052	0.056	0.063	0.052	0.056	0.273	0.052	0.276
			300	0.054	0.055	0.054	0.054	0.049	0.269	0.048	0.272

Notes: See Table 2.5.

Table 7.6. Power properties of the LM-based cointegration test statistics with $h = 3$

B. Type	θ	φ	T	$\tau_{FLM}(1)$	$\tau_{FLM}(2)$	$\tau_{FLM}(3)$	$\tau_{FLM}(\hat{m})$	t_C	ϕ_C	t_N	ϕ_N
No break	0	0	100	0.139	0.090	0.084	0.140	0.196	0.358	0.251	0.394
			200	0.585	0.377	0.253	0.585	0.641	0.927	0.638	0.930
			300	0.872	0.704	0.529	0.872	0.801	0.798	0.866	0.875
	0.4	0	100	0.140	0.091	0.080	0.145	0.211	0.531	0.221	0.474
			200	0.563	0.343	0.232	0.563	0.681	0.428	0.736	0.435
			300	0.878	0.690	0.504	0.878	0.908	0.942	0.918	0.944
	-0.4	0	100	0.146	0.093	0.072	0.146	0.181	0.751	0.199	0.751
			200	0.560	0.394	0.281	0.560	0.487	0.676	0.565	0.733
			300	0.818	0.685	0.543	0.818	0.663	0.991	0.717	0.992
	0	0.4	100	0.141	0.092	0.081	0.146	0.291	0.053	0.272	0.057
			200	0.536	0.333	0.225	0.537	0.748	0.695	0.796	0.702
			300	0.854	0.644	0.463	0.854	0.929	0.987	0.932	0.986
	0	-0.4	100	0.132	0.087	0.076	0.132	0.199	0.254	0.212	0.294
			200	0.534	0.351	0.248	0.534	0.578	0.981	0.607	0.984
			300	0.831	0.682	0.523	0.831	0.735	0.898	0.772	0.928
Sharp	0	0	100	0.122	0.095	0.077	0.123	0.201	0.369	0.197	0.274
			200	0.486	0.331	0.237	0.486	0.642	0.931	0.456	0.833
			300	0.777	0.616	0.477	0.777	0.812	0.804	0.753	0.754
	0.4	0	100	0.132	0.092	0.073	0.136	0.233	0.542	0.189	0.426
			200	0.507	0.324	0.223	0.507	0.675	0.426	0.623	0.312
			300	0.839	0.646	0.488	0.839	0.900	0.932	0.840	0.889
	-0.4	0	100	0.096	0.075	0.066	0.096	0.172	0.741	0.088	0.612
			200	0.371	0.274	0.220	0.371	0.485	0.680	0.328	0.501
			300	0.620	0.523	0.434	0.620	0.660	0.991	0.471	0.975
	0	0.4	100	0.129	0.099	0.079	0.135	0.286	0.062	0.231	0.048
			200	0.505	0.314	0.218	0.507	0.748	0.689	0.704	0.608
			300	0.806	0.605	0.443	0.806	0.933	0.988	0.887	0.974
	0	-0.4	100	0.091	0.077	0.064	0.091	0.190	0.257	0.127	0.152
			200	0.385	0.267	0.208	0.385	0.573	0.978	0.401	0.951
			300	0.656	0.544	0.426	0.656	0.724	0.893	0.566	0.818
Fourier	0	0	100	0.144	0.098	0.084	0.144	0.009	0.000	0.015	0.000
			200	0.582	0.372	0.253	0.582	0.007	0.012	0.005	0.011
			300	0.883	0.702	0.521	0.883	0.006	0.001	0.014	0.001
	0.4	0	100	0.146	0.093	0.080	0.149	0.018	0.015	0.017	0.009
			200	0.591	0.366	0.256	0.591	0.033	0.001	0.044	0.000
			300	0.881	0.692	0.504	0.881	0.077	0.033	0.084	0.035
	-0.4	0	100	0.150	0.107	0.085	0.150	0.014	0.006	0.014	0.006
			200	0.577	0.403	0.289	0.577	0.004	0.000	0.004	0.000
			300	0.824	0.694	0.560	0.824	0.003	0.171	0.004	0.156
	0	0.4	100	0.139	0.089	0.082	0.145	0.044	0.001	0.039	0.001
			200	0.538	0.336	0.225	0.538	0.078	0.017	0.103	0.017
			300	0.861	0.654	0.469	0.861	0.196	0.365	0.195	0.347
	0	-0.4	100	0.141	0.091	0.078	0.141	0.013	0.000	0.012	0.000
			200	0.549	0.360	0.245	0.549	0.005	0.139	0.004	0.131

			300	0.818	0.670	0.518	0.818	0.003	0.000	0.004	0.000
LSTAR	0	0	100	0.153	0.094	0.072	0.153	0.128	0.343	0.177	0.385
			200	0.603	0.417	0.291	0.603	0.409	0.860	0.415	0.875
			300	0.875	0.718	0.554	0.875	0.540	0.719	0.655	0.817
	0.4	0	100	0.148	0.087	0.076	0.149	0.169	0.602	0.175	0.561
			200	0.612	0.399	0.271	0.612	0.503	0.583	0.564	0.599
			300	0.885	0.709	0.544	0.885	0.750	0.935	0.781	0.952
	-0.4	0	100	0.157	0.104	0.087	0.157	0.107	0.606	0.115	0.608
			200	0.629	0.489	0.367	0.629	0.279	0.508	0.341	0.560
			300	0.839	0.753	0.638	0.839	0.378	0.958	0.440	0.967
	0	0.4	100	0.137	0.090	0.084	0.138	0.220	0.110	0.217	0.117
			200	0.576	0.353	0.238	0.576	0.616	0.809	0.670	0.827
			300	0.883	0.689	0.508	0.883	0.825	0.990	0.844	0.991
	0	-0.4	100	0.137	0.093	0.075	0.137	0.120	0.203	0.131	0.239
			200	0.573	0.407	0.291	0.573	0.339	0.928	0.363	0.945
			300	0.829	0.703	0.568	0.829	0.451	0.766	0.506	0.821
ESTAR	0	0	100	0.143	0.101	0.083	0.143	0.191	0.449	0.252	0.480
			200	0.584	0.396	0.266	0.584	0.568	0.942	0.567	0.947
			300	0.877	0.714	0.552	0.877	0.734	0.840	0.826	0.906
	0.4	0	100	0.150	0.105	0.093	0.151	0.209	0.646	0.217	0.601
			200	0.603	0.377	0.257	0.603	0.630	0.683	0.694	0.692
			300	0.877	0.702	0.528	0.877	0.861	0.972	0.876	0.976
	-0.4	0	100	0.154	0.108	0.087	0.154	0.179	0.766	0.187	0.771
			200	0.601	0.462	0.349	0.601	0.426	0.677	0.494	0.727
			300	0.807	0.719	0.601	0.807	0.560	0.988	0.627	0.989
	0	0.4	100	0.144	0.100	0.084	0.146	0.284	0.113	0.264	0.117
			200	0.573	0.348	0.240	0.573	0.697	0.866	0.746	0.876
			300	0.862	0.674	0.494	0.862	0.911	0.996	0.916	0.996
	0	-0.4	100	0.131	0.096	0.075	0.131	0.184	0.287	0.201	0.328
			200	0.549	0.389	0.283	0.549	0.501	0.980	0.539	0.983
			300	0.819	0.690	0.549	0.819	0.646	0.889	0.690	0.920

Notes: See Table 2.5.

Table 7.7. Size properties of the LM-based cointegration test statistics with $h = 4$

B. Type	θ	φ	T	$\tau_{FLM}(1)$	$\tau_{FLM}(2)$	$\tau_{FLM}(3)$	$\tau_{FLM}(\hat{m})$	t_C	ϕ_C	t_N	ϕ_N
No break	0	0	100	0.051	0.050	0.059	0.055	0.062	0.080	0.060	0.080
			200	0.046	0.049	0.057	0.051	0.049	0.054	0.049	0.056
			300	0.043	0.053	0.062	0.050	0.049	0.056	0.049	0.057
	0.4	0	100	0.042	0.048	0.054	0.054	0.061	0.021	0.060	0.022
			200	0.044	0.046	0.052	0.058	0.046	0.012	0.046	0.011
			300	0.051	0.054	0.061	0.063	0.049	0.013	0.049	0.014
	-0.4	0	100	0.047	0.048	0.050	0.047	0.057	0.218	0.058	0.217
			200	0.053	0.057	0.056	0.053	0.051	0.217	0.052	0.213
			300	0.052	0.056	0.054	0.052	0.049	0.218	0.051	0.221
	0	0.4	100	0.049	0.054	0.070	0.066	0.056	0.010	0.058	0.012
			200	0.049	0.052	0.061	0.072	0.055	0.005	0.053	0.005
			300	0.051	0.052	0.059	0.070	0.057	0.006	0.056	0.006
	0	-0.4	100	0.050	0.050	0.049	0.051	0.056	0.152	0.056	0.153
			200	0.047	0.053	0.054	0.047	0.053	0.149	0.053	0.150
			300	0.046	0.048	0.053	0.046	0.045	0.152	0.045	0.152
Sharp	0	0	100	0.042	0.042	0.053	0.049	0.052	0.071	0.055	0.069
			200	0.047	0.047	0.053	0.055	0.052	0.061	0.055	0.060
			300	0.045	0.050	0.056	0.048	0.055	0.060	0.054	0.060
	0.4	0	100	0.043	0.049	0.053	0.055	0.069	0.027	0.064	0.031
			200	0.048	0.047	0.055	0.061	0.055	0.017	0.055	0.015
			300	0.055	0.054	0.057	0.069	0.061	0.018	0.062	0.019
	-0.4	0	100	0.043	0.041	0.044	0.043	0.055	0.200	0.051	0.193
			200	0.048	0.051	0.055	0.048	0.047	0.213	0.046	0.193
			300	0.054	0.046	0.056	0.054	0.052	0.228	0.053	0.218
	0	0.4	100	0.048	0.052	0.060	0.068	0.061	0.011	0.060	0.013
			200	0.050	0.054	0.055	0.070	0.053	0.006	0.054	0.008
			300	0.046	0.047	0.055	0.064	0.054	0.007	0.052	0.007
	0	-0.4	100	0.036	0.046	0.051	0.037	0.052	0.159	0.053	0.144
			200	0.039	0.047	0.053	0.040	0.049	0.145	0.047	0.144
			300	0.051	0.051	0.051	0.052	0.055	0.147	0.055	0.150
Fourier	0	0	100	0.041	0.047	0.059	0.047	0.033	0.019	0.033	0.018
			200	0.044	0.050	0.052	0.049	0.034	0.029	0.035	0.030
			300	0.050	0.045	0.054	0.056	0.037	0.028	0.035	0.029
	0.4	0	100	0.048	0.049	0.057	0.063	0.044	0.010	0.044	0.011
			200	0.044	0.052	0.052	0.057	0.046	0.011	0.047	0.011
			300	0.046	0.051	0.065	0.061	0.043	0.011	0.043	0.011
	-0.4	0	100	0.043	0.051	0.056	0.043	0.020	0.013	0.022	0.014
			200	0.055	0.057	0.059	0.055	0.019	0.046	0.019	0.048
			300	0.050	0.051	0.056	0.050	0.021	0.073	0.021	0.073
	0	0.4	100	0.049	0.051	0.061	0.064	0.047	0.006	0.049	0.006
			200	0.049	0.051	0.061	0.067	0.046	0.006	0.044	0.006
			300	0.051	0.049	0.058	0.066	0.040	0.003	0.039	0.003
	0	-0.4	100	0.044	0.044	0.053	0.045	0.025	0.017	0.026	0.016
			200	0.046	0.050	0.054	0.046	0.023	0.038	0.023	0.040

			300	0.048	0.050	0.055	0.048	0.026	0.063	0.026	0.066
LSTAR	0	0	100	0.046	0.046	0.046	0.048	0.054	0.119	0.055	0.121
			200	0.045	0.050	0.051	0.045	0.048	0.149	0.047	0.152
			300	0.050	0.051	0.059	0.050	0.049	0.180	0.048	0.182
	0.4	0	100	0.048	0.046	0.058	0.053	0.058	0.065	0.059	0.064
			200	0.043	0.051	0.054	0.046	0.048	0.074	0.048	0.076
			300	0.050	0.050	0.059	0.053	0.046	0.084	0.047	0.082
	-0.4	0	100	0.059	0.064	0.057	0.059	0.051	0.232	0.049	0.244
			200	0.082	0.081	0.093	0.082	0.052	0.315	0.051	0.330
			300	0.067	0.076	0.082	0.067	0.044	0.369	0.043	0.383
	0	0.4	100	0.046	0.060	0.061	0.058	0.057	0.040	0.058	0.038
			200	0.046	0.045	0.052	0.054	0.061	0.050	0.059	0.052
			300	0.050	0.054	0.054	0.055	0.055	0.052	0.052	0.053
	0	-0.4	100	0.039	0.043	0.047	0.039	0.054	0.202	0.052	0.208
			200	0.054	0.060	0.062	0.054	0.047	0.260	0.047	0.264
			300	0.060	0.054	0.063	0.060	0.047	0.284	0.048	0.288
ESTAR	0	0	100	0.042	0.048	0.053	0.043	0.062	0.120	0.063	0.123
			200	0.046	0.051	0.054	0.047	0.051	0.144	0.052	0.146
			300	0.049	0.049	0.051	0.049	0.050	0.149	0.049	0.148
	0.4	0	100	0.044	0.047	0.055	0.051	0.062	0.054	0.062	0.051
			200	0.046	0.045	0.055	0.052	0.056	0.069	0.059	0.069
			300	0.051	0.053	0.057	0.058	0.052	0.073	0.051	0.072
	-0.4	0	100	0.051	0.052	0.060	0.051	0.068	0.267	0.063	0.272
			200	0.075	0.080	0.090	0.075	0.063	0.342	0.063	0.344
			300	0.063	0.061	0.066	0.063	0.051	0.342	0.050	0.343
	0	0.4	100	0.045	0.056	0.062	0.056	0.059	0.031	0.058	0.029
			200	0.046	0.054	0.059	0.057	0.054	0.042	0.054	0.040
			300	0.045	0.052	0.059	0.053	0.050	0.036	0.048	0.036
	0	-0.4	100	0.048	0.050	0.051	0.048	0.064	0.209	0.061	0.211
			200	0.055	0.053	0.058	0.055	0.048	0.261	0.051	0.260
			300	0.050	0.055	0.059	0.050	0.046	0.261	0.047	0.266

Notes: See Table 2.5.

Table 7.8. Power properties of the LM-based cointegration test statistics with $h = 4$

B. Type	θ	φ	T	$\tau_{FLM}(1)$	$\tau_{FLM}(2)$	$\tau_{FLM}(3)$	$\tau_{FLM}(\hat{m})$	t_C	ϕ_C	t_N	ϕ_N
No break	0	0	100	0.137	0.094	0.084	0.138	0.178	0.338	0.241	0.367
			200	0.569	0.364	0.251	0.569	0.598	0.906	0.581	0.903
			300	0.874	0.694	0.519	0.874	0.776	0.768	0.847	0.850
	0.4	0	100	0.132	0.089	0.077	0.134	0.212	0.514	0.215	0.456
			200	0.554	0.343	0.225	0.554	0.634	0.396	0.696	0.389
			300	0.873	0.682	0.504	0.873	0.876	0.921	0.893	0.932
	-0.4	0	100	0.136	0.097	0.082	0.136	0.160	0.708	0.175	0.714
			200	0.532	0.370	0.278	0.532	0.431	0.611	0.495	0.658
			300	0.790	0.667	0.528	0.790	0.586	0.982	0.635	0.983
	0	0.4	100	0.134	0.092	0.077	0.140	0.273	0.055	0.250	0.049
			200	0.548	0.335	0.229	0.549	0.728	0.673	0.763	0.681
			300	0.856	0.640	0.468	0.856	0.915	0.980	0.916	0.980
	0	-0.4	100	0.119	0.086	0.073	0.119	0.183	0.225	0.194	0.258
			200	0.504	0.336	0.235	0.504	0.513	0.971	0.545	0.976
			300	0.790	0.643	0.493	0.790	0.669	0.856	0.703	0.887
Sharp	0	0	100	0.116	0.085	0.079	0.116	0.182	0.352	0.178	0.264
			200	0.480	0.325	0.234	0.480	0.603	0.899	0.431	0.817
			300	0.766	0.595	0.458	0.766	0.766	0.757	0.720	0.716
	0.4	0	100	0.128	0.086	0.079	0.131	0.213	0.523	0.174	0.397
			200	0.505	0.319	0.231	0.506	0.650	0.398	0.596	0.307
			300	0.824	0.638	0.479	0.824	0.886	0.923	0.823	0.868
	-0.4	0	100	0.088	0.074	0.068	0.088	0.156	0.702	0.088	0.584
			200	0.358	0.269	0.211	0.358	0.417	0.606	0.293	0.450
			300	0.585	0.502	0.408	0.585	0.592	0.982	0.435	0.963
	0	0.4	100	0.123	0.082	0.072	0.129	0.267	0.046	0.222	0.045
			200	0.495	0.309	0.212	0.496	0.714	0.664	0.672	0.576
			300	0.825	0.614	0.451	0.825	0.922	0.983	0.870	0.966
	0	-0.4	100	0.095	0.073	0.063	0.095	0.179	0.227	0.111	0.136
			200	0.363	0.267	0.200	0.363	0.515	0.969	0.355	0.941
			300	0.627	0.512	0.396	0.627	0.671	0.855	0.519	0.783
Fourier	0	0	100	0.145	0.093	0.080	0.145	0.012	0.001	0.019	0.001
			200	0.582	0.374	0.255	0.582	0.006	0.013	0.005	0.013
			300	0.875	0.702	0.535	0.875	0.006	0.000	0.016	0.001
	0.4	0	100	0.142	0.095	0.088	0.145	0.018	0.020	0.018	0.012
			200	0.570	0.351	0.240	0.571	0.028	0.001	0.040	0.001
			300	0.865	0.683	0.504	0.865	0.083	0.040	0.088	0.043
	-0.4	0	100	0.145	0.099	0.085	0.145	0.013	0.007	0.013	0.006
			200	0.559	0.384	0.281	0.559	0.004	0.000	0.006	0.000
			300	0.787	0.670	0.536	0.787	0.003	0.181	0.003	0.164
	0	0.4	100	0.143	0.094	0.078	0.148	0.043	0.000	0.038	0.000
			200	0.545	0.327	0.223	0.546	0.088	0.019	0.108	0.020
			300	0.852	0.644	0.465	0.852	0.196	0.353	0.191	0.337
	0	-0.4	100	0.136	0.089	0.067	0.136	0.014	0.000	0.013	0.000
			200	0.520	0.350	0.249	0.520	0.007	0.147	0.006	0.137

			300	0.796	0.650	0.500	0.796	0.004	0.002	0.005	0.002
LSTAR	0	0	100	0.145	0.098	0.072	0.145	0.123	0.335	0.166	0.363
			200	0.586	0.389	0.275	0.586	0.396	0.840	0.401	0.850
			300	0.850	0.695	0.543	0.850	0.518	0.683	0.622	0.787
	0.4	0	100	0.143	0.091	0.080	0.144	0.160	0.577	0.159	0.536
			200	0.582	0.387	0.264	0.582	0.469	0.548	0.544	0.563
			300	0.870	0.702	0.527	0.870	0.723	0.926	0.746	0.939
	-0.4	0	100	0.151	0.108	0.089	0.151	0.104	0.574	0.109	0.578
			200	0.575	0.449	0.344	0.575	0.241	0.460	0.300	0.520
			300	0.790	0.707	0.593	0.790	0.348	0.941	0.405	0.947
	0	0.4	100	0.142	0.087	0.075	0.144	0.217	0.107	0.211	0.109
			200	0.549	0.337	0.229	0.549	0.584	0.783	0.645	0.804
			300	0.868	0.671	0.492	0.868	0.804	0.982	0.819	0.986
	0	-0.4	100	0.127	0.090	0.073	0.127	0.106	0.184	0.111	0.209
			200	0.518	0.384	0.274	0.518	0.310	0.912	0.341	0.917
			300	0.780	0.671	0.537	0.780	0.416	0.732	0.464	0.783
ESTAR	0	0	100	0.152	0.095	0.083	0.152	0.179	0.407	0.231	0.443
			200	0.569	0.370	0.264	0.569	0.521	0.925	0.515	0.932
			300	0.837	0.689	0.531	0.837	0.678	0.794	0.776	0.868
	0.4	0	100	0.141	0.092	0.072	0.142	0.199	0.624	0.199	0.567
			200	0.578	0.366	0.259	0.578	0.592	0.647	0.650	0.640
			300	0.874	0.702	0.529	0.874	0.839	0.962	0.861	0.969
	-0.4	0	100	0.136	0.098	0.081	0.136	0.156	0.728	0.170	0.729
			200	0.555	0.412	0.317	0.555	0.367	0.613	0.432	0.670
			300	0.774	0.685	0.573	0.774	0.515	0.980	0.573	0.982
	0	0.4	100	0.147	0.096	0.079	0.149	0.262	0.106	0.253	0.112
			200	0.560	0.345	0.226	0.560	0.686	0.846	0.732	0.856
			300	0.854	0.671	0.487	0.854	0.877	0.993	0.887	0.994
	0	-0.4	100	0.121	0.081	0.076	0.121	0.165	0.244	0.175	0.272
			200	0.516	0.357	0.263	0.516	0.476	0.974	0.501	0.977
			300	0.772	0.654	0.514	0.772	0.577	0.848	0.617	0.880

Notes: See Table 2.5.

Table 7.9. Size properties of the LM-based cointegration test statistics with $h = 5$

B. Type	θ	φ	T	$\tau_{FLM}(1)$	$\tau_{FLM}(2)$	$\tau_{FLM}(3)$	$\tau_{FLM}(\hat{m})$	t_C	ϕ_C	t_N	ϕ_N
No break	0	0	100	0.045	0.054	0.058	0.050	0.061	0.071	0.060	0.069
			200	0.052	0.052	0.056	0.059	0.051	0.058	0.051	0.058
			300	0.045	0.046	0.050	0.049	0.056	0.059	0.055	0.058
	0.4	0	100	0.043	0.046	0.056	0.053	0.062	0.026	0.061	0.025
			200	0.047	0.048	0.053	0.060	0.058	0.017	0.059	0.016
			300	0.051	0.059	0.056	0.063	0.053	0.015	0.055	0.015
	-0.4	0	100	0.049	0.044	0.055	0.049	0.057	0.208	0.058	0.207
			200	0.055	0.055	0.060	0.055	0.051	0.207	0.051	0.205
			300	0.042	0.044	0.054	0.042	0.051	0.220	0.051	0.222
	0	0.4	100	0.052	0.053	0.062	0.059	0.056	0.011	0.058	0.011
			200	0.051	0.052	0.061	0.068	0.053	0.007	0.053	0.006
			300	0.052	0.055	0.061	0.069	0.053	0.008	0.052	0.009
	0	-0.4	100	0.045	0.047	0.054	0.045	0.054	0.142	0.054	0.145
			200	0.049	0.049	0.057	0.049	0.047	0.147	0.046	0.145
			300	0.050	0.048	0.048	0.050	0.050	0.154	0.052	0.157
Sharp	0	0	100	0.040	0.043	0.055	0.050	0.057	0.069	0.055	0.070
			200	0.047	0.054	0.055	0.054	0.050	0.059	0.050	0.055
			300	0.046	0.050	0.059	0.052	0.056	0.064	0.053	0.061
	0.4	0	100	0.045	0.047	0.056	0.055	0.064	0.025	0.060	0.027
			200	0.048	0.052	0.061	0.059	0.058	0.016	0.054	0.018
			300	0.050	0.059	0.063	0.066	0.050	0.012	0.051	0.014
	-0.4	0	100	0.042	0.044	0.051	0.043	0.054	0.194	0.045	0.173
			200	0.057	0.047	0.051	0.057	0.051	0.211	0.050	0.202
			300	0.047	0.049	0.059	0.047	0.050	0.220	0.049	0.214
	0	0.4	100	0.045	0.046	0.060	0.063	0.061	0.011	0.062	0.013
			200	0.049	0.053	0.057	0.064	0.054	0.007	0.056	0.009
			300	0.051	0.053	0.060	0.067	0.050	0.005	0.051	0.005
	0	-0.4	100	0.045	0.047	0.049	0.047	0.053	0.150	0.054	0.143
			200	0.043	0.049	0.052	0.050	0.046	0.150	0.050	0.136
			300	0.055	0.054	0.053	0.048	0.049	0.146	0.049	0.148
Fourier	0	0	100	0.043	0.047	0.053	0.047	0.036	0.020	0.035	0.020
			200	0.043	0.052	0.056	0.048	0.032	0.022	0.032	0.022
			300	0.047	0.050	0.057	0.052	0.034	0.030	0.033	0.029
	0.4	0	100	0.045	0.051	0.060	0.054	0.045	0.010	0.046	0.010
			200	0.041	0.052	0.054	0.055	0.043	0.010	0.043	0.009
			300	0.054	0.051	0.054	0.067	0.043	0.011	0.044	0.010
	-0.4	0	100	0.046	0.046	0.058	0.045	0.028	0.015	0.029	0.016
			200	0.050	0.056	0.050	0.050	0.019	0.042	0.019	0.042
			300	0.044	0.048	0.050	0.045	0.021	0.069	0.021	0.069
	0	0.4	100	0.051	0.052	0.072	0.061	0.049	0.007	0.050	0.006
			200	0.049	0.051	0.058	0.068	0.043	0.006	0.043	0.006
			300	0.051	0.046	0.058	0.074	0.040	0.005	0.041	0.004
	0	-0.4	100	0.041	0.044	0.051	0.042	0.028	0.016	0.027	0.018
			200	0.048	0.048	0.048	0.049	0.024	0.041	0.024	0.040

			300	0.046	0.047	0.047	0.047	0.027	0.060	0.026	0.060
LSTAR	0	0	100	0.046	0.049	0.050	0.048	0.051	0.117	0.050	0.117
			200	0.045	0.051	0.050	0.045	0.053	0.146	0.050	0.145
			300	0.053	0.046	0.055	0.053	0.044	0.170	0.049	0.169
	0.4	0	100	0.043	0.048	0.055	0.049	0.055	0.064	0.056	0.062
			200	0.050	0.047	0.055	0.052	0.057	0.083	0.057	0.084
			300	0.050	0.056	0.063	0.052	0.049	0.088	0.047	0.087
	-0.4	0	100	0.058	0.053	0.052	0.058	0.052	0.230	0.050	0.237
			200	0.079	0.079	0.088	0.069	0.053	0.305	0.053	0.306
			300	0.073	0.079	0.090	0.073	0.052	0.359	0.053	0.373
	0	0.4	100	0.049	0.052	0.062	0.062	0.063	0.039	0.063	0.037
			200	0.047	0.050	0.062	0.053	0.048	0.043	0.048	0.042
			300	0.053	0.056	0.055	0.058	0.051	0.052	0.051	0.054
	0	-0.4	100	0.045	0.044	0.053	0.045	0.057	0.208	0.059	0.213
			200	0.058	0.056	0.064	0.058	0.051	0.232	0.051	0.244
			300	0.049	0.055	0.055	0.049	0.048	0.281	0.048	0.285
ESTAR	0	0	100	0.044	0.047	0.052	0.044	0.057	0.109	0.058	0.113
			200	0.047	0.053	0.055	0.047	0.049	0.140	0.046	0.141
			300	0.048	0.051	0.060	0.048	0.046	0.140	0.047	0.136
	0.4	0	100	0.045	0.054	0.059	0.052	0.058	0.052	0.057	0.052
			200	0.047	0.049	0.054	0.052	0.053	0.071	0.056	0.073
			300	0.051	0.053	0.056	0.055	0.047	0.068	0.049	0.070
	-0.4	0	100	0.047	0.057	0.057	0.047	0.052	0.234	0.051	0.238
			200	0.075	0.075	0.079	0.075	0.061	0.342	0.060	0.340
			300	0.066	0.064	0.072	0.066	0.052	0.343	0.052	0.348
	0	0.4	100	0.049	0.051	0.065	0.060	0.063	0.036	0.064	0.034
			200	0.044	0.046	0.058	0.053	0.052	0.041	0.050	0.040
			300	0.048	0.052	0.056	0.056	0.050	0.036	0.051	0.037
	0	-0.4	100	0.045	0.049	0.051	0.045	0.060	0.198	0.059	0.204
			200	0.049	0.050	0.053	0.049	0.056	0.258	0.058	0.266
			300	0.049	0.060	0.061	0.049	0.051	0.264	0.051	0.268

Notes: See Table 2.5.

Table 7.10. Power properties of the LM-based cointegration test statistics with $h = 5$

B. Type	θ	φ	T	$\tau_{FLM}(1)$	$\tau_{FLM}(2)$	$\tau_{FLM}(3)$	$\tau_{FLM}(\hat{m})$	t_C	ϕ_C	t_N	ϕ_N
No break	0	0	100	0.133	0.090	0.078	0.134	0.170	0.326	0.233	0.350
			200	0.555	0.359	0.250	0.555	0.540	0.874	0.531	0.873
			300	0.847	0.666	0.506	0.847	0.730	0.718	0.807	0.810
	0.4	0	100	0.131	0.085	0.074	0.133	0.195	0.485	0.198	0.424
			200	0.554	0.334	0.225	0.554	0.611	0.373	0.673	0.367
			300	0.863	0.680	0.502	0.863	0.864	0.901	0.872	0.909
	-0.4	0	100	0.130	0.094	0.075	0.130	0.146	0.660	0.150	0.663
			200	0.506	0.362	0.265	0.506	0.378	0.571	0.440	0.614
			300	0.746	0.638	0.511	0.746	0.536	0.972	0.588	0.977
	0	0.4	100	0.136	0.095	0.077	0.140	0.258	0.051	0.244	0.050
			200	0.536	0.321	0.223	0.537	0.689	0.632	0.738	0.645
			300	0.839	0.630	0.451	0.839	0.904	0.977	0.907	0.977
	0	-0.4	100	0.121	0.086	0.070	0.121	0.156	0.194	0.157	0.222
			200	0.486	0.321	0.228	0.486	0.467	0.950	0.493	0.953
			300	0.756	0.619	0.480	0.756	0.619	0.811	0.656	0.849
Sharp	0	0	100	0.123	0.082	0.080	0.124	0.168	0.312	0.161	0.236
			200	0.462	0.303	0.219	0.462	0.554	0.878	0.403	0.793
			300	0.739	0.589	0.458	0.739	0.721	0.714	0.687	0.684
	0.4	0	100	0.120	0.088	0.076	0.123	0.195	0.488	0.162	0.372
			200	0.489	0.315	0.223	0.490	0.607	0.375	0.573	0.269
			300	0.811	0.639	0.478	0.811	0.863	0.903	0.797	0.847
	-0.4	0	100	0.089	0.068	0.064	0.089	0.141	0.671	0.088	0.552
			200	0.339	0.269	0.208	0.339	0.383	0.546	0.262	0.401
			300	0.566	0.478	0.389	0.566	0.520	0.973	0.402	0.951
	0	0.4	100	0.133	0.092	0.080	0.138	0.260	0.049	0.221	0.046
			200	0.504	0.304	0.214	0.507	0.694	0.638	0.665	0.566
			300	0.808	0.615	0.447	0.808	0.910	0.978	0.854	0.963
	0	-0.4	100	0.091	0.064	0.068	0.091	0.165	0.196	0.107	0.126
			200	0.349	0.257	0.191	0.349	0.471	0.956	0.341	0.917
			300	0.593	0.478	0.386	0.593	0.600	0.806	0.463	0.722
Fourier	0	0	100	0.149	0.096	0.078	0.150	0.011	0.001	0.019	0.001
			200	0.565	0.376	0.247	0.565	0.009	0.017	0.007	0.016
			300	0.851	0.683	0.519	0.851	0.007	0.000	0.020	0.002
	0.4	0	100	0.142	0.095	0.080	0.143	0.022	0.021	0.021	0.016
			200	0.566	0.353	0.237	0.566	0.032	0.001	0.042	0.001
			300	0.869	0.681	0.511	0.869	0.081	0.043	0.086	0.045
	-0.4	0	100	0.143	0.098	0.083	0.143	0.016	0.008	0.015	0.007
			200	0.505	0.369	0.259	0.505	0.003	0.000	0.006	0.000
			300	0.764	0.642	0.516	0.764	0.003	0.171	0.004	0.159
	0	0.4	100	0.131	0.096	0.086	0.137	0.044	0.001	0.040	0.000
			200	0.538	0.324	0.221	0.539	0.085	0.021	0.105	0.021
			300	0.845	0.641	0.461	0.845	0.199	0.355	0.195	0.341
	0	-0.4	100	0.133	0.084	0.072	0.133	0.016	0.000	0.016	0.000
			200	0.500	0.343	0.241	0.500	0.009	0.158	0.008	0.151

			300	0.761	0.628	0.475	0.761	0.004	0.001	0.005	0.001
LSTAR	0	0	100	0.138	0.097	0.084	0.138	0.106	0.315	0.148	0.337
			200	0.554	0.370	0.266	0.554	0.357	0.815	0.356	0.815
			300	0.837	0.682	0.530	0.837	0.501	0.653	0.600	0.764
	0.4	0	100	0.147	0.093	0.084	0.148	0.159	0.556	0.161	0.512
			200	0.565	0.366	0.251	0.565	0.460	0.532	0.530	0.547
			300	0.862	0.693	0.530	0.862	0.688	0.910	0.729	0.921
	-0.4	0	100	0.144	0.097	0.089	0.144	0.098	0.560	0.104	0.564
			200	0.534	0.417	0.324	0.534	0.212	0.414	0.265	0.469
			300	0.745	0.660	0.560	0.745	0.318	0.928	0.381	0.936
	0	0.4	100	0.142	0.095	0.083	0.144	0.218	0.107	0.213	0.114
			200	0.550	0.346	0.237	0.550	0.561	0.765	0.615	0.784
			300	0.854	0.668	0.497	0.854	0.776	0.983	0.793	0.986
	0	-0.4	100	0.122	0.089	0.075	0.122	0.108	0.166	0.115	0.195
			200	0.491	0.362	0.269	0.491	0.283	0.895	0.314	0.906
			300	0.746	0.632	0.510	0.746	0.369	0.692	0.421	0.741
ESTAR	0	0	100	0.131	0.089	0.076	0.131	0.158	0.384	0.208	0.407
			200	0.546	0.366	0.257	0.546	0.480	0.904	0.480	0.907
			300	0.819	0.674	0.519	0.819	0.631	0.753	0.731	0.843
	0.4	0	100	0.135	0.091	0.087	0.135	0.188	0.605	0.190	0.551
			200	0.562	0.362	0.251	0.562	0.550	0.600	0.616	0.603
			300	0.846	0.671	0.506	0.846	0.804	0.947	0.819	0.952
	-0.4	0	100	0.124	0.083	0.079	0.124	0.143	0.674	0.148	0.677
			200	0.521	0.402	0.302	0.521	0.325	0.562	0.392	0.604
			300	0.730	0.645	0.545	0.730	0.468	0.970	0.529	0.970
	0	0.4	100	0.139	0.092	0.086	0.142	0.247	0.103	0.235	0.105
			200	0.549	0.347	0.244	0.549	0.661	0.825	0.712	0.837
			300	0.850	0.657	0.483	0.850	0.872	0.992	0.877	0.993
	0	-0.4	100	0.123	0.095	0.077	0.123	0.149	0.208	0.155	0.239
			200	0.479	0.343	0.255	0.479	0.421	0.957	0.445	0.960
			300	0.738	0.600	0.483	0.738	0.533	0.815	0.579	0.846

Notes: See Table 2.5.

7.2. Appendix B

Table 7.11. Response surface estimates with no lag for the $\tilde{\tau}_{FLM}$ statistic with cumulative Fourier frequencies

m	$\theta \backslash l$	0.01	0.05	0.10
1	Intercept	-4.5330	-4.0175	-3.7605
	$1/T$	-14.3470	-10.8832	-6.6518
	$(1/T)^2$	103.5749	597.0086	344.8498
	$(1/T)^3$	-6723.9384	-31306.3550	-17598.6627
	$(1/T)^4$	99465.7565	481696.9562	255940.9110
1,2	Intercept	-5.2716	-4.7696	-4.5086
	$1/T$	-23.0769	-11.7332	-9.7982
	$(1/T)^2$	445.0858	1.8112	229.6720
	$(1/T)^3$	-24538.3505	-4232.0800	-15932.2701
	$(1/T)^4$	294365.8882	50336.1418	252455.1990
1,2,3	Intercept	-5.8887	-5.3927	-5.1294
	$1/T$	-23.6339	-13.0670	-11.0585
	$(1/T)^2$	-571.9403	-539.0685	-351.8827
	$(1/T)^3$	13325.9441	13272.2052	9769.3417
	$(1/T)^4$	-249139.2879	-179693.1826	-151116.5936

Notes: *m* is the cumulative frequencies, *l* is the significance level, and *T* is the sample size.

Table 7.12. Response surface estimates with lags for the $\tilde{\tau}_{FLM}$ statistic with cumulative Fourier frequencies

m	$\theta \backslash l$	0.01	0.05	0.10
1	Intercept	-4.5400	-4.0354	-3.7708
	$1/T$	-9.9526	-6.0576	-4.7598
	$(1/T)^2$	-125.2395	-46.4292	-9.0986
	p/T	-0.4216	0.1348	0.2213
	$(p/T)^2$	1.7619	-0.5618	-0.9175
1,2	Intercept	-5.2745	-4.7755	-4.5129
	$1/T$	-18.9003	-10.2906	-6.9191
	$(1/T)^2$	-160.2890	-121.6433	-94.6904
	p/T	-0.0280	0.0783	-0.0345
	$(p/T)^2$	0.3545	-0.8065	-0.2241
1,2,3	Intercept	-5.8891	-5.3837	-5.1277
	$1/T$	-25.8949	-16.0727	-12.8118
	$(1/T)^2$	-380.7933	-232.9316	-162.6554
	p/T	0.2738	-0.0836	-0.0187
	$(p/T)^2$	-0.4696	0.7699	0.5217

Notes: See Table 7.9.

Table 7.13. Size properties of panel statistics with $h = 2$

B. Type	θ	φ	N	T	$P(1)$	$P_M(1)$	$P(2)$	$P_M(2)$	$P(\hat{m}_i)$	$P_M(\hat{m}_i)$	$Z_\tau(1)$	$Z_\tau(2)$	Z_τ	Z_ϕ
No break	0	0	20	100	0.055	0.063	0.040	0.050	0.056	0.065	0.083	0.110	0.097	0.063
			20	200	0.048	0.057	0.043	0.052	0.056	0.064	0.068	0.068	0.076	0.061
			40	100	0.044	0.049	0.031	0.035	0.046	0.055	0.074	0.128	0.100	0.053
			40	200	0.038	0.046	0.033	0.039	0.050	0.055	0.055	0.075	0.072	0.049
	0.3	0	20	100	0.038	0.045	0.035	0.045	0.044	0.051	0.052	0.104	0.091	0.062
			20	200	0.035	0.042	0.040	0.049	0.054	0.065	0.051	0.075	0.063	0.039
			40	100	0.028	0.036	0.034	0.041	0.037	0.040	0.043	0.127	0.092	0.049
			40	200	0.041	0.047	0.040	0.044	0.065	0.076	0.056	0.082	0.064	0.033
	-0.3	0	20	100	0.030	0.037	0.031	0.038	0.028	0.036	0.045	0.073	0.078	0.173
			20	200	0.044	0.052	0.046	0.052	0.044	0.052	0.059	0.070	0.073	0.193
			40	100	0.027	0.031	0.020	0.024	0.024	0.028	0.037	0.071	0.091	0.235
			40	200	0.031	0.037	0.035	0.040	0.031	0.037	0.046	0.065	0.068	0.247
	0	0.3	20	100	0.054	0.062	0.045	0.058	0.068	0.085	0.084	0.122	0.059	0.004
			20	200	0.040	0.050	0.043	0.051	0.071	0.082	0.057	0.084	0.061	0.002
			40	100	0.049	0.056	0.038	0.044	0.057	0.069	0.087	0.161	0.062	0.001
			40	200	0.041	0.046	0.037	0.042	0.078	0.089	0.062	0.087	0.045	0.001
	0	-0.3	20	100	0.040	0.047	0.025	0.031	0.040	0.046	0.059	0.073	0.063	0.156
			20	200	0.047	0.056	0.036	0.045	0.048	0.056	0.061	0.061	0.060	0.199
			40	100	0.030	0.033	0.018	0.021	0.028	0.032	0.053	0.076	0.067	0.210
			40	200	0.035	0.043	0.026	0.031	0.037	0.043	0.055	0.057	0.051	0.271
Sharp	0	0	20	100	0.052	0.059	0.049	0.057	0.056	0.067	0.094	0.123	0.089	0.058
			20	200	0.047	0.058	0.040	0.047	0.062	0.073	0.073	0.073	0.075	0.056
			40	100	0.051	0.057	0.036	0.042	0.049	0.056	0.086	0.130	0.111	0.065
			40	200	0.045	0.050	0.033	0.038	0.051	0.059	0.067	0.074	0.082	0.060
	0.3	0	20	100	0.041	0.050	0.050	0.059	0.041	0.048	0.065	0.119	0.087	0.061
			20	200	0.042	0.052	0.046	0.056	0.062	0.074	0.059	0.082	0.059	0.037
			40	100	0.027	0.031	0.026	0.030	0.025	0.028	0.039	0.113	0.083	0.043
			40	200	0.030	0.036	0.037	0.044	0.051	0.058	0.045	0.089	0.065	0.030
	-0.3	0	20	100	0.035	0.044	0.032	0.041	0.034	0.042	0.048	0.087	0.092	0.194
			20	200	0.045	0.055	0.044	0.050	0.045	0.056	0.062	0.076	0.074	0.204
			40	100	0.026	0.032	0.023	0.029	0.023	0.029	0.037	0.067	0.085	0.275
			40	200	0.037	0.045	0.039	0.044	0.041	0.048	0.052	0.072	0.067	0.276
	0	0.3	20	100	0.049	0.059	0.044	0.051	0.065	0.078	0.082	0.129	0.073	0.002
			20	200	0.043	0.051	0.038	0.047	0.071	0.082	0.063	0.077	0.056	0.001
			40	100	0.037	0.042	0.039	0.045	0.049	0.054	0.080	0.152	0.070	0.001
			40	200	0.037	0.046	0.032	0.036	0.073	0.081	0.066	0.064	0.055	0.000
	0	-0.3	20	100	0.041	0.048	0.032	0.036	0.032	0.040	0.071	0.077	0.067	0.177
			20	200	0.040	0.049	0.025	0.034	0.041	0.050	0.063	0.051	0.069	0.210
			40	100	0.030	0.038	0.017	0.019	0.027	0.033	0.055	0.067	0.062	0.238
			40	200	0.035	0.039	0.023	0.026	0.038	0.042	0.052	0.050	0.053	0.290
Smooth	0	0	20	100	0.046	0.056	0.040	0.050	0.052	0.057	0.086	0.102	0.018	0.011
			20	200	0.042	0.051	0.043	0.048	0.053	0.064	0.066	0.071	0.000	0.000
			40	100	0.045	0.050	0.031	0.039	0.042	0.048	0.080	0.121	0.093	0.074
			40	200	0.038	0.044	0.032	0.037	0.046	0.054	0.057	0.076	0.000	0.000

	0.3	0	20	100	0.031	0.038	0.043	0.050	0.035	0.040	0.052	0.103	0.025	0.019
			20	200	0.044	0.052	0.042	0.052	0.058	0.071	0.055	0.075	0.000	0.000
			40	100	0.033	0.036	0.031	0.037	0.037	0.041	0.048	0.103	0.135	0.112
			40	200	0.035	0.040	0.030	0.038	0.051	0.061	0.049	0.072	0.000	0.000
	-0.3	0	20	100	0.030	0.037	0.026	0.033	0.028	0.034	0.043	0.067	0.000	0.001
			20	200	0.039	0.048	0.039	0.047	0.039	0.048	0.050	0.065	0.000	0.000
			40	100	0.028	0.032	0.014	0.016	0.027	0.030	0.035	0.054	0.001	0.002
			40	200	0.033	0.039	0.030	0.037	0.033	0.039	0.050	0.068	0.000	0.000
	0	0.3	20	100	0.054	0.062	0.046	0.053	0.066	0.080	0.090	0.120	0.038	0.007
			20	200	0.040	0.052	0.042	0.050	0.071	0.084	0.067	0.073	0.000	0.000
			40	100	0.042	0.049	0.038	0.043	0.065	0.074	0.094	0.152	0.191	0.040
			40	200	0.039	0.047	0.034	0.040	0.079	0.090	0.061	0.079	0.000	0.000
	0	-0.3	20	100	0.039	0.045	0.022	0.026	0.038	0.045	0.067	0.067	0.000	0.001
			20	200	0.047	0.059	0.041	0.049	0.046	0.060	0.072	0.069	0.000	0.000
			40	100	0.023	0.028	0.014	0.019	0.022	0.026	0.050	0.059	0.003	0.006
			40	200	0.036	0.040	0.024	0.028	0.038	0.043	0.050	0.050	0.000	0.000
Mixed	0	0	20	100	0.047	0.058	0.037	0.050	0.055	0.069	0.081	0.116	0.000	0.000
			20	200	0.049	0.058	0.033	0.040	0.057	0.065	0.066	0.065	0.001	0.001
			40	100	0.040	0.046	0.027	0.033	0.037	0.043	0.079	0.128	0.000	0.000
			40	200	0.043	0.048	0.032	0.036	0.054	0.061	0.067	0.072	0.000	0.000
	0.3	0	20	100	0.032	0.041	0.033	0.041	0.044	0.051	0.049	0.096	0.001	0.001
			20	200	0.035	0.042	0.043	0.051	0.054	0.065	0.046	0.076	0.004	0.004
			40	100	0.027	0.032	0.030	0.038	0.034	0.041	0.046	0.118	0.000	0.000
			40	200	0.036	0.043	0.039	0.044	0.054	0.064	0.050	0.081	0.001	0.000
	-0.3	0	20	100	0.026	0.032	0.025	0.029	0.024	0.030	0.039	0.072	0.000	0.001
			20	200	0.036	0.042	0.036	0.044	0.037	0.041	0.051	0.064	0.000	0.001
			40	100	0.016	0.021	0.015	0.019	0.017	0.021	0.032	0.064	0.000	0.000
			40	200	0.032	0.039	0.031	0.038	0.033	0.039	0.048	0.057	0.000	0.000
	0	0.3	20	100	0.043	0.049	0.039	0.047	0.062	0.072	0.070	0.115	0.000	0.000
			20	200	0.043	0.056	0.038	0.044	0.068	0.083	0.061	0.072	0.006	0.000
			40	100	0.040	0.046	0.030	0.035	0.053	0.062	0.076	0.139	0.000	0.000
			40	200	0.024	0.027	0.032	0.035	0.054	0.069	0.043	0.065	0.001	0.000
	0	-0.3	20	100	0.032	0.039	0.021	0.026	0.032	0.040	0.054	0.065	0.000	0.001
			20	200	0.036	0.045	0.037	0.041	0.039	0.048	0.054	0.054	0.000	0.002
			40	100	0.021	0.026	0.017	0.020	0.021	0.026	0.050	0.069	0.000	0.000
			40	200	0.033	0.037	0.017	0.020	0.035	0.039	0.051	0.040	0.000	0.001

Notes: See Table 3.2.

Table 7.14. Power properties of panel statistics with $h = 2$

B. Type	θ	φ	N	T	$P(1)$	$P_M(1)$	$P(2)$	$P_M(2)$	$P(\hat{m}_i)$	$P_M(\hat{m}_i)$	$Z_\tau(1)$	$Z_\tau(2)$	Z_τ	Z_ϕ
No break	0	0	20	100	0.549	0.581	0.205	0.231	0.524	0.552	0.768	0.456	0.946	0.932
			20	200	1.000	1.000	0.951	0.962	1.000	1.000	1.000	0.994	1.000	1.000
			40	100	0.812	0.830	0.306	0.331	0.767	0.785	0.961	0.687	0.998	0.997
			40	200	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.3	0	20	100	0.272	0.303	0.110	0.131	0.259	0.286	0.419	0.260	0.903	0.886
			20	200	0.993	0.994	0.761	0.793	0.992	0.994	0.998	0.905	1.000	1.000
			40	100	0.402	0.432	0.125	0.143	0.372	0.402	0.599	0.391	0.997	0.995
			40	200	1.000	1.000	0.954	0.960	1.000	1.000	1.000	0.995	1.000	1.000
	-0.3	0	20	100	0.270	0.304	0.092	0.106	0.252	0.286	0.406	0.220	0.541	0.787
			20	200	0.996	0.996	0.803	0.827	0.995	0.995	1.000	0.926	0.999	1.000
			40	100	0.417	0.447	0.105	0.121	0.378	0.405	0.614	0.318	0.884	0.982
			40	200	1.000	1.000	0.970	0.975	1.000	1.000	1.000	0.997	1.000	1.000
	0	0.3	20	100	0.510	0.552	0.195	0.221	0.509	0.539	0.730	0.452	0.984	0.775
			20	200	1.000	1.000	0.939	0.951	1.000	1.000	1.000	0.991	1.000	1.000
			40	100	0.768	0.789	0.293	0.321	0.731	0.750	0.944	0.683	1.000	0.971
			40	200	1.000	1.000	0.998	0.998	1.000	1.000	1.000	1.000	1.000	1.000
	0	-0.3	20	100	0.495	0.535	0.162	0.188	0.470	0.507	0.717	0.391	0.753	0.933
			20	200	1.000	1.000	0.960	0.967	0.999	0.999	1.000	0.994	1.000	1.000
			40	100	0.773	0.797	0.234	0.256	0.722	0.748	0.939	0.596	0.980	0.999
			40	200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Sharp	0	0	20	100	0.531	0.564	0.213	0.238	0.484	0.515	0.738	0.447	0.971	0.963
			20	200	1.000	1.000	0.930	0.943	0.999	0.999	1.000	0.987	1.000	1.000
			40	100	0.809	0.828	0.313	0.340	0.734	0.755	0.945	0.687	1.000	0.999
			40	200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.3	0	20	100	0.255	0.284	0.109	0.127	0.235	0.263	0.397	0.262	0.929	0.936
			20	200	0.985	0.988	0.714	0.744	0.983	0.988	0.999	0.880	1.000	1.000
			40	100	0.403	0.432	0.137	0.154	0.357	0.382	0.609	0.379	0.997	0.995
			40	200	1.000	1.000	0.946	0.956	1.000	1.000	1.000	0.994	1.000	1.000
	-0.3	0	20	100	0.253	0.285	0.101	0.117	0.229	0.255	0.391	0.231	0.758	0.929
			20	200	0.937	0.946	0.643	0.677	0.935	0.944	0.981	0.835	1.000	1.000
			40	100	0.396	0.430	0.106	0.124	0.354	0.379	0.592	0.311	0.974	0.998
			40	200	1.000	1.000	0.965	0.971	0.999	0.999	1.000	0.995	1.000	1.000
	0	0.3	20	100	0.476	0.517	0.188	0.211	0.445	0.478	0.695	0.434	0.986	0.847
			20	200	1.000	1.000	0.897	0.912	0.998	0.999	1.000	0.976	1.000	1.000
			40	100	0.740	0.762	0.282	0.306	0.677	0.699	0.919	0.661	1.000	0.983
			40	200	1.000	1.000	0.997	0.998	1.000	1.000	1.000	1.000	1.000	1.000
	0	-0.3	20	100	0.483	0.522	0.164	0.192	0.436	0.473	0.694	0.381	0.918	0.984
			20	200	0.995	0.997	0.857	0.878	0.994	0.996	1.000	0.958	1.000	1.000
			40	100	0.722	0.747	0.211	0.235	0.647	0.671	0.915	0.568	0.995	0.999
			40	200	1.000	1.000	0.998	0.999	1.000	1.000	1.000	1.000	1.000	1.000
Smooth	0	0	20	100	0.556	0.591	0.199	0.225	0.516	0.555	0.768	0.445	0.003	0.003
			20	200	1.000	1.000	0.959	0.965	1.000	1.000	1.000	0.994	0.002	0.007
			40	100	0.805	0.826	0.295	0.318	0.760	0.781	0.953	0.664	0.005	0.005
			40	200	1.000	1.000	0.999	0.999	1.000	1.000	1.000	1.000	0.000	0.000

	0.3	0	20	100	0.255	0.289	0.093	0.106	0.239	0.272	0.404	0.239	0.003	0.002
			20	200	0.992	0.994	0.748	0.778	0.992	0.994	1.000	0.899	0.007	0.011
			40	100	0.412	0.447	0.132	0.144	0.379	0.408	0.636	0.372	0.003	0.003
			40	200	1.000	1.000	0.960	0.964	1.000	1.000	1.000	0.997	0.001	0.002
	-0.3	0	20	100	0.273	0.301	0.083	0.101	0.252	0.282	0.404	0.201	0.000	0.000
			20	200	0.997	0.997	0.793	0.824	0.996	0.996	1.000	0.931	0.000	0.001
			40	100	0.428	0.457	0.084	0.098	0.392	0.421	0.627	0.271	0.000	0.000
			40	200	1.000	1.000	0.970	0.973	1.000	1.000	1.000	0.997	0.000	0.000
	0	0.3	20	100	0.503	0.540	0.194	0.219	0.484	0.514	0.722	0.438	0.006	0.002
			20	200	1.000	1.000	0.940	0.951	1.000	1.000	1.000	0.990	0.041	0.016
			40	100	0.753	0.775	0.286	0.310	0.719	0.743	0.938	0.678	0.008	0.007
			40	200	1.000	1.000	0.998	0.998	1.000	1.000	1.000	1.000	0.006	0.001
	0	-0.3	20	100	0.499	0.537	0.153	0.176	0.465	0.502	0.704	0.359	0.000	0.001
			20	200	1.000	1.000	0.955	0.961	0.999	1.000	1.000	0.995	0.000	0.002
			40	100	0.773	0.794	0.198	0.218	0.718	0.738	0.942	0.529	0.000	0.000
			40	200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.000	0.000
Mixed	0	0	20	100	0.478	0.513	0.189	0.214	0.451	0.488	0.707	0.430	0.005	0.034
			20	200	0.999	0.999	0.938	0.945	0.999	0.999	1.000	0.990	0.843	0.932
			40	100	0.729	0.754	0.279	0.299	0.682	0.712	0.925	0.633	0.005	0.047
			40	200	1.000	1.000	0.999	0.999	1.000	1.000	1.000	1.000	0.997	1.000
	0.3	0	20	100	0.253	0.284	0.100	0.115	0.238	0.272	0.392	0.253	0.018	0.047
			20	200	0.987	0.990	0.716	0.748	0.985	0.988	0.998	0.883	0.912	0.959
			40	100	0.388	0.417	0.129	0.145	0.354	0.386	0.590	0.364	0.033	0.074
			40	200	1.000	1.000	0.945	0.951	1.000	1.000	1.000	0.993	0.998	1.000
	-0.3	0	20	100	0.198	0.223	0.068	0.081	0.183	0.205	0.315	0.189	0.000	0.021
			20	200	0.979	0.985	0.702	0.738	0.978	0.984	0.997	0.875	0.067	0.502
			40	100	0.287	0.314	0.082	0.092	0.265	0.289	0.468	0.278	0.000	0.019
			40	200	1.000	1.000	0.938	0.944	0.999	0.999	1.000	0.990	0.113	0.888
	0	0.3	20	100	0.433	0.470	0.157	0.179	0.431	0.471	0.670	0.402	0.049	0.009
			20	200	1.000	1.000	0.904	0.918	1.000	1.000	1.000	0.984	0.998	0.956
			40	100	0.691	0.713	0.240	0.262	0.661	0.685	0.904	0.612	0.106	0.018
			40	200	1.000	1.000	0.996	0.998	1.000	1.000	1.000	1.000	1.000	1.000
	0	-0.3	20	100	0.376	0.413	0.122	0.145	0.352	0.384	0.589	0.316	0.000	0.042
			20	200	0.999	0.999	0.900	0.914	0.998	0.998	1.000	0.982	0.320	0.864
			40	100	0.583	0.609	0.151	0.167	0.530	0.559	0.836	0.443	0.000	0.043
			40	200	1.000	1.000	0.995	0.996	1.000	1.000	1.000	1.000	0.677	0.996

Notes: See Table 3.2.

Table 7.15. Size properties of panel statistics with $h = 3$

B. Type	θ	φ	N	T	$P(1)$	$P_M(1)$	$P(2)$	$P_M(2)$	$P(\hat{m}_i)$	$P_M(\hat{m}_i)$	$Z_\tau(1)$	$Z_\tau(2)$	Z_τ	Z_ϕ
No break	0	0	20	100	0.055	0.064	0.043	0.053	0.055	0.069	0.093	0.123	0.103	0.066
			20	200	0.056	0.063	0.039	0.047	0.065	0.075	0.074	0.062	0.080	0.060
			40	100	0.045	0.051	0.034	0.040	0.044	0.052	0.080	0.129	0.106	0.056
			40	200	0.042	0.048	0.033	0.037	0.050	0.059	0.063	0.074	0.070	0.050
	0.3	0	20	100	0.036	0.048	0.048	0.059	0.039	0.052	0.060	0.118	0.090	0.058
			20	200	0.035	0.040	0.039	0.045	0.055	0.061	0.049	0.066	0.062	0.035
			40	100	0.027	0.032	0.037	0.042	0.038	0.041	0.044	0.130	0.087	0.041
			40	200	0.032	0.035	0.036	0.042	0.052	0.060	0.044	0.077	0.073	0.035
	-0.3	0	20	100	0.036	0.045	0.028	0.034	0.034	0.042	0.052	0.074	0.088	0.190
			20	200	0.049	0.062	0.050	0.058	0.050	0.062	0.058	0.080	0.074	0.194
			40	100	0.027	0.031	0.022	0.025	0.023	0.026	0.037	0.075	0.090	0.250
			40	200	0.041	0.046	0.034	0.039	0.044	0.049	0.054	0.060	0.071	0.259
	0	0.3	20	100	0.051	0.060	0.056	0.065	0.066	0.078	0.081	0.143	0.069	0.003
			20	200	0.048	0.057	0.044	0.052	0.067	0.082	0.067	0.078	0.053	0.002
			40	100	0.040	0.046	0.042	0.046	0.062	0.068	0.081	0.174	0.056	0.000
			40	200	0.040	0.048	0.039	0.046	0.078	0.087	0.062	0.085	0.042	0.000
	0	-0.3	20	100	0.039	0.047	0.031	0.036	0.036	0.042	0.065	0.077	0.067	0.168
			20	200	0.043	0.050	0.037	0.043	0.043	0.051	0.063	0.067	0.056	0.187
			40	100	0.032	0.037	0.020	0.023	0.032	0.036	0.060	0.071	0.059	0.213
			40	200	0.039	0.045	0.023	0.027	0.039	0.047	0.059	0.046	0.052	0.271
Sharp	0	0	20	100	0.064	0.071	0.043	0.053	0.060	0.070	0.097	0.120	0.095	0.064
			20	200	0.054	0.061	0.048	0.056	0.058	0.067	0.075	0.083	0.066	0.055
			40	100	0.043	0.050	0.037	0.042	0.043	0.053	0.085	0.132	0.098	0.051
			40	200	0.045	0.049	0.036	0.044	0.057	0.067	0.066	0.076	0.078	0.056
	0.3	0	20	100	0.042	0.054	0.048	0.056	0.049	0.057	0.060	0.117	0.086	0.059
			20	200	0.042	0.052	0.049	0.059	0.059	0.070	0.056	0.085	0.081	0.053
			40	100	0.031	0.035	0.033	0.038	0.031	0.035	0.042	0.110	0.089	0.050
			40	200	0.035	0.041	0.046	0.053	0.062	0.070	0.048	0.086	0.068	0.033
	-0.3	0	20	100	0.038	0.044	0.028	0.036	0.035	0.039	0.054	0.076	0.081	0.187
			20	200	0.043	0.051	0.042	0.050	0.045	0.052	0.058	0.069	0.079	0.193
			40	100	0.024	0.029	0.018	0.022	0.023	0.026	0.036	0.069	0.091	0.256
			40	200	0.036	0.040	0.032	0.038	0.037	0.041	0.054	0.063	0.067	0.259
	0	0.3	20	100	0.050	0.060	0.043	0.051	0.056	0.066	0.090	0.122	0.066	0.003
			20	200	0.040	0.052	0.047	0.057	0.073	0.090	0.065	0.077	0.060	0.004
			40	100	0.043	0.050	0.037	0.043	0.050	0.057	0.080	0.149	0.062	0.000
			40	200	0.034	0.041	0.033	0.038	0.069	0.078	0.056	0.071	0.040	0.000
	0	-0.3	20	100	0.036	0.043	0.027	0.032	0.038	0.046	0.061	0.077	0.061	0.169
			20	200	0.041	0.049	0.034	0.039	0.044	0.051	0.053	0.063	0.064	0.205
			40	100	0.023	0.027	0.015	0.020	0.022	0.025	0.049	0.067	0.071	0.267
			40	200	0.038	0.044	0.027	0.030	0.040	0.046	0.055	0.055	0.047	0.295
Smooth	0	0	20	100	0.050	0.061	0.031	0.038	0.052	0.064	0.083	0.095	0.013	0.007
			20	200	0.046	0.055	0.036	0.045	0.054	0.064	0.065	0.067	0.000	0.000
			40	100	0.046	0.053	0.036	0.041	0.050	0.059	0.083	0.112	0.112	0.087
			40	200	0.041	0.047	0.030	0.036	0.047	0.056	0.065	0.066	0.000	0.000

	0.3	0	20	100	0.032	0.042	0.039	0.045	0.040	0.048	0.049	0.102	0.027	0.022
			20	200	0.036	0.046	0.039	0.048	0.056	0.070	0.051	0.068	0.000	0.000
			40	100	0.030	0.036	0.039	0.044	0.038	0.042	0.051	0.126	0.125	0.105
			40	200	0.032	0.038	0.040	0.046	0.055	0.062	0.047	0.078	0.000	0.000
	-0.3	0	20	100	0.038	0.048	0.029	0.033	0.036	0.048	0.055	0.071	0.000	0.000
			20	200	0.042	0.049	0.038	0.048	0.042	0.049	0.055	0.065	0.000	0.000
			40	100	0.029	0.034	0.016	0.019	0.025	0.028	0.041	0.056	0.002	0.002
			40	200	0.032	0.039	0.034	0.039	0.033	0.040	0.050	0.063	0.000	0.000
	0	0.3	20	100	0.050	0.060	0.045	0.055	0.072	0.083	0.083	0.124	0.030	0.003
			20	200	0.052	0.059	0.047	0.053	0.071	0.081	0.071	0.076	0.000	0.000
			40	100	0.035	0.042	0.038	0.043	0.061	0.071	0.077	0.152	0.198	0.037
			40	200	0.040	0.044	0.040	0.044	0.073	0.083	0.062	0.079	0.000	0.000
	0	-0.3	20	100	0.043	0.050	0.025	0.032	0.041	0.049	0.068	0.067	0.000	0.000
			20	200	0.046	0.055	0.031	0.038	0.046	0.056	0.066	0.056	0.000	0.000
			40	100	0.027	0.034	0.017	0.018	0.027	0.034	0.052	0.059	0.002	0.006
			40	200	0.034	0.039	0.024	0.029	0.035	0.040	0.057	0.054	0.000	0.000
Mixed	0	0	20	100	0.044	0.053	0.038	0.042	0.050	0.063	0.082	0.105	0.000	0.000
			20	200	0.050	0.057	0.044	0.054	0.057	0.066	0.066	0.079	0.001	0.002
			40	100	0.040	0.048	0.032	0.038	0.049	0.056	0.087	0.120	0.000	0.000
			40	200	0.043	0.048	0.033	0.040	0.055	0.061	0.059	0.067	0.000	0.000
	0.3	0	20	100	0.036	0.044	0.036	0.049	0.042	0.051	0.052	0.111	0.001	0.000
			20	200	0.037	0.048	0.046	0.054	0.061	0.072	0.055	0.079	0.003	0.002
			40	100	0.024	0.029	0.033	0.037	0.027	0.034	0.041	0.114	0.000	0.000
			40	200	0.035	0.041	0.035	0.041	0.049	0.059	0.052	0.071	0.002	0.001
	-0.3	0	20	100	0.027	0.030	0.029	0.035	0.026	0.029	0.041	0.072	0.000	0.001
			20	200	0.039	0.048	0.041	0.051	0.040	0.049	0.053	0.069	0.000	0.002
			40	100	0.020	0.024	0.024	0.027	0.020	0.023	0.032	0.071	0.000	0.000
			40	200	0.031	0.036	0.031	0.038	0.031	0.035	0.039	0.065	0.000	0.000
	0	0.3	20	100	0.041	0.051	0.042	0.053	0.058	0.068	0.069	0.113	0.001	0.000
			20	200	0.042	0.047	0.040	0.049	0.064	0.075	0.060	0.069	0.004	0.000
			40	100	0.033	0.039	0.031	0.033	0.053	0.061	0.068	0.138	0.000	0.000
			40	200	0.032	0.035	0.031	0.035	0.059	0.069	0.046	0.064	0.001	0.000
	0	-0.3	20	100	0.034	0.038	0.021	0.025	0.032	0.036	0.055	0.067	0.000	0.000
			20	200	0.039	0.047	0.029	0.034	0.041	0.049	0.053	0.048	0.000	0.002
			40	100	0.021	0.024	0.012	0.015	0.021	0.025	0.041	0.052	0.000	0.000
			40	200	0.026	0.029	0.013	0.016	0.027	0.030	0.040	0.032	0.000	0.000

Notes: See Table 3.2.

Table 7.16. Power properties of panel statistics with $h = 3$

B. Type	θ	φ	N	T	$P(1)$	$P_M(1)$	$P(2)$	$P_M(2)$	$P(\hat{m}_i)$	$P_M(\hat{m}_i)$	$Z_\tau(1)$	$Z_\tau(2)$	Z_τ	Z_ϕ
No break	0	0	20	100	0.576	0.609	0.223	0.253	0.543	0.577	0.772	0.479	0.947	0.938
			20	200	1.000	1.000	0.952	0.960	1.000	1.000	1.000	0.993	1.000	1.000
			40	100	0.808	0.831	0.304	0.324	0.760	0.782	0.960	0.684	0.999	0.999
			40	200	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.3	0	20	100	0.259	0.295	0.107	0.119	0.257	0.287	0.418	0.254	0.897	0.892
			20	200	0.996	0.996	0.764	0.789	0.995	0.996	0.999	0.913	1.000	1.000
			40	100	0.415	0.440	0.127	0.141	0.370	0.394	0.620	0.379	0.997	0.995
			40	200	1.000	1.000	0.954	0.959	1.000	1.000	1.000	0.996	1.000	1.000
	-0.3	0	20	100	0.271	0.305	0.092	0.107	0.251	0.284	0.409	0.228	0.643	0.821
			20	200	0.997	0.998	0.814	0.838	0.995	0.996	0.999	0.939	0.999	1.000
			40	100	0.418	0.445	0.117	0.129	0.387	0.414	0.607	0.331	0.858	0.967
			40	200	1.000	1.000	0.971	0.973	1.000	1.000	1.000	0.997	1.000	1.000
	0	0.3	20	100	0.518	0.555	0.199	0.226	0.498	0.539	0.731	0.453	0.979	0.787
			20	200	1.000	1.000	0.935	0.948	1.000	1.000	1.000	0.989	1.000	1.000
			40	100	0.774	0.799	0.307	0.337	0.754	0.776	0.949	0.711	1.000	0.980
			40	200	1.000	1.000	0.997	0.998	1.000	1.000	1.000	1.000	1.000	1.000
	0	-0.3	20	100	0.486	0.523	0.169	0.191	0.452	0.492	0.704	0.382	0.790	0.946
			20	200	1.000	1.000	0.957	0.967	0.999	0.999	1.000	0.994	1.000	1.000
			40	100	0.760	0.781	0.237	0.256	0.713	0.733	0.937	0.605	0.977	0.997
			40	200	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Sharp	0	0	20	100	0.544	0.581	0.214	0.238	0.495	0.532	0.753	0.458	0.982	0.977
			20	200	1.000	1.000	0.925	0.940	0.999	0.999	1.000	0.990	1.000	1.000
			40	100	0.794	0.816	0.313	0.339	0.721	0.743	0.944	0.685	1.000	1.000
			40	200	1.000	1.000	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000
	0.3	0	20	100	0.273	0.306	0.110	0.126	0.255	0.279	0.411	0.258	0.920	0.920
			20	200	0.985	0.987	0.729	0.752	0.984	0.987	0.998	0.889	1.000	1.000
			40	100	0.415	0.445	0.131	0.148	0.365	0.390	0.621	0.383	0.998	0.998
			40	200	1.000	1.000	0.950	0.956	1.000	1.000	1.000	0.995	1.000	1.000
	-0.3	0	20	100	0.262	0.290	0.098	0.116	0.231	0.259	0.380	0.231	0.774	0.931
			20	200	0.945	0.955	0.660	0.692	0.943	0.953	0.982	0.839	0.999	1.000
			40	100	0.392	0.420	0.113	0.124	0.350	0.373	0.583	0.316	0.967	0.996
			40	200	1.000	1.000	0.967	0.972	0.999	0.999	1.000	0.997	1.000	1.000
	0	0.3	20	100	0.474	0.514	0.191	0.220	0.443	0.479	0.705	0.448	0.985	0.813
			20	200	0.999	0.999	0.901	0.912	1.000	1.000	1.000	0.980	1.000	1.000
			40	100	0.739	0.762	0.286	0.308	0.684	0.704	0.927	0.659	1.000	0.983
			40	200	1.000	1.000	0.998	0.998	1.000	1.000	1.000	1.000	1.000	1.000
	0	-0.3	20	100	0.478	0.512	0.158	0.186	0.441	0.478	0.686	0.382	0.914	0.983
			20	200	0.995	0.997	0.838	0.863	0.993	0.997	0.999	0.955	1.000	1.000
			40	100	0.737	0.764	0.239	0.256	0.666	0.692	0.917	0.582	0.996	0.999
			40	200	1.000	1.000	0.997	0.998	1.000	1.000	1.000	1.000	1.000	1.000
Smooth	0	0	20	100	0.556	0.594	0.204	0.229	0.522	0.560	0.766	0.447	0.003	0.003
			20	200	1.000	1.000	0.953	0.964	0.999	0.999	1.000	0.994	0.002	0.004
			40	100	0.811	0.833	0.285	0.309	0.765	0.788	0.963	0.655	0.004	0.004
			40	200	1.000	1.000	0.999	0.999	1.000	1.000	1.000	1.000	0.000	0.000

	0.3	0	20	100	0.277	0.309	0.105	0.121	0.253	0.281	0.413	0.236	0.004	0.005
			20	200	0.990	0.992	0.752	0.782	0.989	0.991	0.999	0.904	0.004	0.011
			40	100	0.422	0.450	0.126	0.144	0.381	0.410	0.628	0.359	0.002	0.002
			40	200	1.000	1.000	0.957	0.963	1.000	1.000	1.000	0.997	0.000	0.001
	-0.3	0	20	100	0.260	0.296	0.080	0.094	0.242	0.274	0.403	0.198	0.000	0.000
			20	200	0.996	0.997	0.781	0.806	0.994	0.994	1.000	0.927	0.000	0.001
			40	100	0.420	0.454	0.099	0.115	0.387	0.417	0.632	0.289	0.000	0.000
			40	200	1.000	1.000	0.968	0.972	1.000	1.000	1.000	0.998	0.000	0.000
	0	0.3	20	100	0.501	0.534	0.197	0.220	0.477	0.518	0.712	0.429	0.006	0.004
			20	200	1.000	1.000	0.935	0.945	1.000	1.000	1.000	0.986	0.042	0.012
			40	100	0.760	0.782	0.285	0.314	0.721	0.746	0.935	0.673	0.007	0.006
			40	200	1.000	1.000	0.997	0.999	1.000	1.000	1.000	1.000	0.007	0.002
	0	-0.3	20	100	0.511	0.549	0.157	0.180	0.470	0.506	0.728	0.364	0.000	0.001
			20	200	1.000	1.000	0.946	0.954	1.000	1.000	1.000	0.994	0.000	0.002
			40	100	0.751	0.772	0.202	0.224	0.686	0.712	0.934	0.529	0.000	0.000
			40	200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.000	0.000
Mixed	0	0	20	100	0.485	0.521	0.184	0.208	0.464	0.499	0.688	0.416	0.009	0.031
			20	200	1.000	1.000	0.937	0.948	0.999	1.000	1.000	0.990	0.844	0.930
			40	100	0.751	0.777	0.270	0.294	0.703	0.725	0.931	0.638	0.004	0.047
			40	200	1.000	1.000	0.998	0.999	1.000	1.000	1.000	1.000	0.997	0.999
	0.3	0	20	100	0.247	0.280	0.105	0.122	0.231	0.259	0.383	0.254	0.029	0.070
			20	200	0.984	0.988	0.720	0.749	0.983	0.989	0.998	0.879	0.878	0.950
			40	100	0.364	0.392	0.111	0.126	0.334	0.360	0.576	0.369	0.034	0.085
			40	200	0.999	1.000	0.947	0.955	1.000	1.000	1.000	0.991	0.997	0.999
	-0.3	0	20	100	0.184	0.212	0.066	0.083	0.170	0.195	0.303	0.194	0.000	0.013
			20	200	0.979	0.984	0.706	0.734	0.977	0.982	0.996	0.878	0.073	0.571
			40	100	0.295	0.320	0.083	0.094	0.267	0.293	0.491	0.271	0.000	0.020
			40	200	1.000	1.000	0.922	0.931	0.999	0.999	1.000	0.988	0.135	0.891
	0	0.3	20	100	0.424	0.461	0.151	0.177	0.420	0.455	0.644	0.379	0.070	0.012
			20	200	1.000	1.000	0.923	0.934	1.000	1.000	1.000	0.985	0.996	0.945
			40	100	0.710	0.735	0.248	0.275	0.664	0.685	0.905	0.633	0.095	0.010
			40	200	1.000	1.000	0.998	0.999	1.000	1.000	1.000	1.000	1.000	0.999
	0	-0.3	20	100	0.371	0.405	0.117	0.134	0.343	0.374	0.593	0.304	0.000	0.037
			20	200	1.000	1.000	0.900	0.918	0.998	0.999	1.000	0.981	0.338	0.880
			40	100	0.606	0.632	0.158	0.181	0.560	0.585	0.844	0.470	0.000	0.060
			40	200	1.000	1.000	0.994	0.995	1.000	1.000	1.000	1.000	0.642	0.996

Notes: See Table 3.2.

Table 7.17. Size properties of panel statistics with $h = 4$

B. Type	θ	φ	N	T	$P(1)$	$P_M(1)$	$P(2)$	$P_M(2)$	$P(\hat{m}_i)$	$P_M(\hat{m}_i)$	$Z_\tau(1)$	$Z_\tau(2)$	Z_τ	Z_ϕ
No break	0	0	20	100	0.056	0.067	0.043	0.050	0.056	0.064	0.090	0.119	0.098	0.061
			20	200	0.055	0.064	0.047	0.056	0.063	0.077	0.073	0.081	0.075	0.063
			40	100	0.044	0.053	0.035	0.038	0.041	0.047	0.086	0.132	0.100	0.056
			40	200	0.037	0.042	0.027	0.030	0.048	0.058	0.062	0.066	0.083	0.059
	0.3	0	20	100	0.038	0.045	0.038	0.046	0.042	0.052	0.056	0.109	0.095	0.055
			20	200	0.039	0.049	0.044	0.053	0.054	0.068	0.055	0.082	0.074	0.049
			40	100	0.024	0.028	0.029	0.034	0.034	0.041	0.042	0.130	0.093	0.048
			40	200	0.030	0.035	0.040	0.046	0.050	0.057	0.046	0.080	0.066	0.029
	-0.3	0	20	100	0.030	0.036	0.028	0.035	0.028	0.033	0.044	0.070	0.089	0.178
			20	200	0.041	0.050	0.047	0.058	0.042	0.051	0.058	0.074	0.068	0.182
			40	100	0.025	0.029	0.020	0.022	0.020	0.024	0.034	0.072	0.082	0.236
			40	200	0.029	0.034	0.032	0.037	0.031	0.036	0.046	0.067	0.066	0.244
	0	0.3	20	100	0.046	0.055	0.052	0.060	0.068	0.080	0.083	0.126	0.061	0.004
			20	200	0.044	0.052	0.046	0.056	0.070	0.083	0.065	0.086	0.063	0.002
			40	100	0.044	0.048	0.043	0.049	0.060	0.068	0.082	0.171	0.061	0.000
			40	200	0.045	0.055	0.037	0.043	0.090	0.103	0.074	0.081	0.052	0.001
	0	-0.3	20	100	0.043	0.052	0.033	0.038	0.042	0.051	0.067	0.081	0.065	0.170
			20	200	0.047	0.056	0.041	0.048	0.048	0.057	0.068	0.067	0.065	0.216
			40	100	0.029	0.036	0.019	0.023	0.029	0.033	0.060	0.079	0.059	0.221
			40	200	0.034	0.040	0.024	0.029	0.036	0.041	0.051	0.057	0.046	0.276
Sharp	0	0	20	100	0.050	0.060	0.049	0.058	0.047	0.055	0.089	0.127	0.093	0.061
			20	200	0.048	0.056	0.041	0.050	0.058	0.069	0.073	0.075	0.074	0.058
			40	100	0.041	0.046	0.035	0.041	0.042	0.045	0.081	0.135	0.095	0.056
			40	200	0.045	0.052	0.039	0.044	0.056	0.065	0.067	0.083	0.072	0.053
	0.3	0	20	100	0.045	0.055	0.040	0.047	0.052	0.062	0.065	0.106	0.083	0.054
			20	200	0.042	0.050	0.040	0.049	0.062	0.071	0.058	0.075	0.074	0.047
			40	100	0.031	0.035	0.032	0.037	0.032	0.037	0.049	0.124	0.091	0.048
			40	200	0.034	0.041	0.035	0.039	0.055	0.065	0.047	0.071	0.067	0.030
	-0.3	0	20	100	0.030	0.038	0.030	0.037	0.028	0.033	0.044	0.077	0.081	0.181
			20	200	0.048	0.056	0.042	0.053	0.049	0.057	0.061	0.070	0.069	0.187
			40	100	0.025	0.029	0.024	0.028	0.025	0.027	0.037	0.075	0.092	0.275
			40	200	0.038	0.046	0.035	0.039	0.041	0.047	0.053	0.069	0.077	0.264
	0	0.3	20	100	0.050	0.060	0.041	0.049	0.065	0.081	0.090	0.123	0.073	0.005
			20	200	0.047	0.054	0.038	0.044	0.059	0.072	0.064	0.066	0.058	0.002
			40	100	0.038	0.047	0.040	0.044	0.050	0.056	0.086	0.151	0.072	0.000
			40	200	0.035	0.040	0.030	0.038	0.066	0.077	0.058	0.078	0.051	0.001
	0	-0.3	20	100	0.045	0.053	0.029	0.033	0.042	0.048	0.067	0.080	0.070	0.189
			20	200	0.037	0.045	0.032	0.037	0.039	0.047	0.057	0.053	0.061	0.205
			40	100	0.029	0.033	0.018	0.021	0.023	0.026	0.048	0.075	0.068	0.254
			40	200	0.031	0.036	0.023	0.028	0.036	0.040	0.052	0.047	0.055	0.275
Smooth	0	0	20	100	0.052	0.062	0.049	0.055	0.053	0.065	0.087	0.107	0.012	0.009
			20	200	0.046	0.054	0.041	0.050	0.049	0.063	0.065	0.069	0.000	0.000
			40	100	0.042	0.049	0.031	0.036	0.045	0.052	0.081	0.101	0.103	0.076
			40	200	0.035	0.041	0.030	0.034	0.043	0.049	0.056	0.068	0.000	0.000

	0.3	0	20	100	0.044	0.050	0.040	0.047	0.046	0.055	0.065	0.094	0.029	0.019
			20	200	0.038	0.046	0.038	0.048	0.048	0.059	0.051	0.073	0.000	0.000
			40	100	0.030	0.034	0.028	0.034	0.035	0.040	0.050	0.106	0.134	0.108
			40	200	0.039	0.044	0.037	0.043	0.068	0.075	0.053	0.079	0.000	0.000
	-0.3	0	20	100	0.031	0.038	0.024	0.032	0.030	0.036	0.046	0.067	0.000	0.000
			20	200	0.036	0.045	0.040	0.049	0.037	0.046	0.055	0.068	0.000	0.000
			40	100	0.024	0.028	0.014	0.018	0.022	0.027	0.034	0.054	0.005	0.006
			40	200	0.036	0.045	0.040	0.043	0.037	0.046	0.050	0.073	0.000	0.000
	0	0.3	20	100	0.048	0.057	0.044	0.051	0.068	0.079	0.081	0.130	0.038	0.006
			20	200	0.045	0.056	0.035	0.042	0.079	0.092	0.062	0.064	0.000	0.000
			40	100	0.043	0.048	0.039	0.045	0.067	0.075	0.088	0.154	0.192	0.035
			40	200	0.039	0.043	0.037	0.046	0.070	0.082	0.064	0.083	0.000	0.000
	0	-0.3	20	100	0.038	0.044	0.022	0.025	0.035	0.040	0.065	0.065	0.001	0.002
			20	200	0.044	0.051	0.036	0.041	0.045	0.052	0.058	0.062	0.000	0.000
			40	100	0.028	0.031	0.015	0.017	0.024	0.028	0.047	0.056	0.002	0.004
			40	200	0.033	0.039	0.025	0.029	0.036	0.041	0.054	0.050	0.000	0.000
Mixed	0	0	20	100	0.047	0.058	0.050	0.059	0.053	0.065	0.087	0.125	0.000	0.000
			20	200	0.051	0.059	0.050	0.058	0.061	0.072	0.071	0.085	0.002	0.003
			40	100	0.044	0.050	0.033	0.039	0.042	0.048	0.084	0.125	0.000	0.000
			40	200	0.044	0.050	0.033	0.038	0.056	0.063	0.070	0.078	0.000	0.000
	0.3	0	20	100	0.040	0.050	0.043	0.055	0.043	0.053	0.057	0.114	0.001	0.001
			20	200	0.039	0.051	0.040	0.049	0.058	0.071	0.057	0.084	0.004	0.002
			40	100	0.026	0.031	0.033	0.039	0.033	0.037	0.037	0.119	0.000	0.000
			40	200	0.033	0.037	0.042	0.048	0.052	0.060	0.041	0.080	0.001	0.001
	-0.3	0	20	100	0.030	0.038	0.030	0.037	0.031	0.036	0.052	0.076	0.000	0.001
			20	200	0.046	0.054	0.046	0.051	0.045	0.054	0.061	0.071	0.000	0.001
			40	100	0.013	0.017	0.016	0.020	0.013	0.016	0.028	0.064	0.000	0.000
			40	200	0.032	0.036	0.032	0.035	0.032	0.036	0.044	0.067	0.000	0.001
	0	0.3	20	100	0.043	0.051	0.033	0.040	0.055	0.066	0.070	0.098	0.001	0.000
			20	200	0.037	0.046	0.035	0.040	0.061	0.076	0.056	0.067	0.005	0.000
			40	100	0.036	0.040	0.031	0.035	0.049	0.056	0.075	0.129	0.000	0.000
			40	200	0.035	0.041	0.029	0.035	0.063	0.071	0.056	0.067	0.001	0.000
	0	-0.3	20	100	0.032	0.039	0.019	0.024	0.029	0.033	0.053	0.065	0.000	0.001
			20	200	0.042	0.048	0.030	0.035	0.044	0.049	0.053	0.047	0.000	0.003
			40	100	0.023	0.028	0.013	0.014	0.019	0.025	0.044	0.060	0.000	0.000
			40	200	0.034	0.037	0.016	0.021	0.036	0.040	0.046	0.039	0.000	0.000

Notes: See Table 3.2.

Table 7.18. Power properties of panel statistics with $h = 4$

B. Type	θ	φ	N	T	$P(1)$	$P_M(1)$	$P(2)$	$P_M(2)$	$P(\hat{m}_i)$	$P_M(\hat{m}_i)$	$Z_\tau(1)$	$Z_\tau(2)$	Z_τ	Z_ϕ
No break	0	0	20	100	0.573	0.602	0.226	0.256	0.535	0.568	0.770	0.474	0.954	0.937
			20	200	1.000	1.000	0.955	0.965	1.000	1.000	1.000	0.995	1.000	1.000
			40	100	0.812	0.831	0.313	0.340	0.764	0.784	0.951	0.702	1.000	0.999
			40	200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.3	0	20	100	0.261	0.293	0.109	0.126	0.258	0.286	0.408	0.260	0.904	0.904
			20	200	0.993	0.993	0.748	0.772	0.992	0.993	0.999	0.901	1.000	1.000
			40	100	0.416	0.440	0.138	0.154	0.388	0.413	0.623	0.394	0.994	0.993
			40	200	1.000	1.000	0.958	0.961	1.000	1.000	1.000	0.996	1.000	1.000
	-0.3	0	20	100	0.265	0.298	0.099	0.117	0.245	0.274	0.403	0.234	0.556	0.789
			20	200	0.995	0.996	0.797	0.824	0.994	0.995	0.999	0.925	0.999	1.000
			40	100	0.424	0.453	0.116	0.127	0.388	0.415	0.616	0.316	0.879	0.978
			40	200	1.000	1.000	0.971	0.973	1.000	1.000	1.000	0.998	1.000	1.000
	0	0.3	20	100	0.513	0.546	0.211	0.238	0.509	0.543	0.724	0.461	0.983	0.807
			20	200	1.000	1.000	0.937	0.950	1.000	1.000	1.000	0.986	1.000	1.000
			40	100	0.779	0.803	0.295	0.316	0.751	0.773	0.938	0.697	0.999	0.966
			40	200	1.000	1.000	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000
	0	-0.3	20	100	0.514	0.553	0.187	0.208	0.489	0.528	0.730	0.408	0.847	0.967
			20	200	1.000	1.000	0.955	0.965	1.000	1.000	1.000	0.995	1.000	1.000
			40	100	0.780	0.801	0.249	0.272	0.727	0.750	0.944	0.603	0.970	0.997
			40	200	1.000	1.000	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000
Sharp	0	0	20	100	0.532	0.574	0.227	0.254	0.493	0.535	0.739	0.457	0.979	0.970
			20	200	0.999	0.999	0.938	0.949	0.998	0.999	1.000	0.992	1.000	1.000
			40	100	0.798	0.819	0.319	0.342	0.734	0.753	0.953	0.680	1.000	1.000
			40	200	1.000	1.000	0.998	0.998	1.000	1.000	1.000	1.000	1.000	1.000
	0.3	0	20	100	0.269	0.300	0.122	0.136	0.238	0.263	0.402	0.261	0.935	0.935
			20	200	0.986	0.989	0.713	0.742	0.985	0.988	0.997	0.885	1.000	1.000
			40	100	0.411	0.440	0.133	0.150	0.353	0.381	0.606	0.375	0.999	0.999
			40	200	1.000	1.000	0.951	0.959	1.000	1.000	1.000	0.996	1.000	1.000
	-0.3	0	20	100	0.254	0.284	0.091	0.103	0.223	0.249	0.377	0.213	0.735	0.915
			20	200	0.936	0.944	0.655	0.685	0.934	0.942	0.978	0.829	1.000	1.000
			40	100	0.394	0.421	0.101	0.114	0.358	0.383	0.586	0.319	0.967	0.999
			40	200	1.000	1.000	0.960	0.965	0.999	0.999	1.000	0.996	1.000	1.000
	0	0.3	20	100	0.482	0.517	0.199	0.232	0.443	0.473	0.709	0.447	0.989	0.865
			20	200	1.000	1.000	0.903	0.919	0.998	0.998	1.000	0.979	1.000	1.000
			40	100	0.741	0.763	0.295	0.317	0.678	0.704	0.919	0.674	1.000	0.988
			40	200	1.000	1.000	0.998	0.998	1.000	1.000	1.000	1.000	1.000	1.000
	0	-0.3	20	100	0.467	0.504	0.165	0.189	0.422	0.456	0.688	0.370	0.920	0.981
			20	200	0.991	0.993	0.845	0.865	0.991	0.992	1.000	0.957	1.000	1.000
			40	100	0.735	0.755	0.223	0.246	0.665	0.689	0.921	0.572	0.996	1.000
			40	200	1.000	1.000	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000
Smooth	0	0	20	100	0.540	0.576	0.201	0.233	0.513	0.546	0.760	0.441	0.002	0.002
			20	200	1.000	1.000	0.947	0.955	1.000	1.000	1.000	0.992	0.000	0.004
			40	100	0.808	0.827	0.287	0.307	0.759	0.781	0.958	0.672	0.005	0.004
			40	200	1.000	1.000	0.999	0.999	1.000	1.000	1.000	1.000	0.000	0.000

	0.3	0	20	100	0.267	0.303	0.103	0.122	0.246	0.272	0.415	0.262	0.001	0.002
			20	200	0.993	0.995	0.762	0.788	0.993	0.995	0.999	0.908	0.005	0.009
			40	100	0.421	0.449	0.119	0.137	0.382	0.405	0.639	0.365	0.001	0.002
			40	200	1.000	1.000	0.953	0.960	1.000	1.000	1.000	0.997	0.000	0.000
	-0.3	0	20	100	0.261	0.291	0.082	0.097	0.240	0.271	0.392	0.188	0.000	0.000
			20	200	0.996	0.996	0.788	0.815	0.995	0.995	0.999	0.927	0.000	0.000
			40	100	0.420	0.450	0.091	0.103	0.388	0.416	0.610	0.272	0.000	0.000
			40	200	1.000	1.000	0.973	0.977	1.000	1.000	1.000	0.997	0.000	0.000
	0	0.3	20	100	0.508	0.541	0.205	0.228	0.495	0.532	0.725	0.441	0.007	0.004
			20	200	1.000	1.000	0.928	0.943	1.000	1.000	1.000	0.989	0.041	0.016
			40	100	0.771	0.787	0.288	0.314	0.733	0.754	0.935	0.674	0.004	0.004
			40	200	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	0.005	0.001
	0	-0.3	20	100	0.502	0.542	0.146	0.169	0.470	0.508	0.724	0.360	0.000	0.001
			20	200	1.000	1.000	0.956	0.964	1.000	1.000	1.000	0.993	0.000	0.002
			40	100	0.760	0.784	0.198	0.222	0.708	0.729	0.942	0.539	0.000	0.000
			40	200	1.000	1.000	0.998	0.999	1.000	1.000	1.000	1.000	0.000	0.000
Mixed	0	0	20	100	0.494	0.527	0.191	0.217	0.465	0.495	0.709	0.434	0.007	0.052
			20	200	1.000	1.000	0.938	0.950	1.000	1.000	1.000	0.993	0.869	0.950
			40	100	0.750	0.774	0.275	0.298	0.698	0.723	0.929	0.652	0.004	0.067
			40	200	1.000	1.000	0.998	0.999	1.000	1.000	1.000	1.000	0.996	1.000
	0.3	0	20	100	0.253	0.284	0.102	0.122	0.237	0.267	0.383	0.262	0.027	0.058
			20	200	0.988	0.990	0.724	0.756	0.989	0.991	0.999	0.894	0.903	0.959
			40	100	0.392	0.419	0.129	0.145	0.366	0.391	0.590	0.367	0.034	0.077
			40	200	1.000	1.000	0.947	0.956	1.000	1.000	1.000	0.995	0.995	0.999
	-0.3	0	20	100	0.188	0.214	0.063	0.073	0.174	0.198	0.308	0.180	0.000	0.013
			20	200	0.982	0.985	0.694	0.723	0.980	0.983	0.996	0.868	0.065	0.518
			40	100	0.289	0.315	0.074	0.088	0.265	0.292	0.484	0.263	0.001	0.020
			40	200	1.000	1.000	0.929	0.942	0.999	0.999	1.000	0.993	0.124	0.890
	0	0.3	20	100	0.433	0.471	0.168	0.192	0.423	0.457	0.654	0.399	0.070	0.017
			20	200	0.999	0.999	0.907	0.920	0.999	0.999	1.000	0.980	0.996	0.949
			40	100	0.692	0.714	0.233	0.254	0.654	0.676	0.898	0.607	0.094	0.011
			40	200	1.000	1.000	0.997	0.997	1.000	1.000	1.000	1.000	1.000	1.000
	0	-0.3	20	100	0.367	0.404	0.113	0.128	0.345	0.379	0.587	0.304	0.000	0.024
			20	200	0.999	1.000	0.911	0.929	0.997	0.997	1.000	0.984	0.341	0.891
			40	100	0.603	0.630	0.160	0.172	0.557	0.582	0.841	0.464	0.000	0.058
			40	200	1.000	1.000	0.995	0.995	1.000	1.000	1.000	1.000	0.599	0.993

Notes: See Table 3.2.

Table 7.19. Size properties of panel statistics with $h = 5$

B. Type	θ	φ	N	T	$P(1)$	$P_M(1)$	$P(2)$	$P_M(2)$	$P(\hat{m}_i)$	$P_M(\hat{m}_i)$	$Z_\tau(1)$	$Z_\tau(2)$	Z_τ	Z_ϕ
No break	0	0	20	100	0.050	0.060	0.044	0.052	0.054	0.064	0.088	0.120	0.100	0.067
			20	200	0.046	0.056	0.040	0.047	0.052	0.067	0.062	0.071	0.066	0.054
			40	100	0.040	0.050	0.039	0.043	0.046	0.053	0.083	0.125	0.099	0.057
			40	200	0.044	0.052	0.034	0.041	0.053	0.061	0.071	0.077	0.080	0.057
	0.3	0	20	100	0.036	0.048	0.039	0.047	0.043	0.053	0.054	0.106	0.089	0.058
			20	200	0.037	0.044	0.044	0.055	0.057	0.068	0.050	0.078	0.073	0.046
			40	100	0.030	0.034	0.032	0.037	0.032	0.041	0.049	0.125	0.091	0.041
			40	200	0.031	0.037	0.035	0.040	0.050	0.060	0.044	0.084	0.069	0.033
	-0.3	0	20	100	0.034	0.043	0.027	0.035	0.032	0.040	0.052	0.079	0.091	0.174
			20	200	0.049	0.058	0.039	0.051	0.049	0.058	0.055	0.067	0.079	0.198
			40	100	0.025	0.031	0.020	0.023	0.024	0.027	0.034	0.074	0.088	0.242
			40	200	0.033	0.040	0.034	0.038	0.033	0.039	0.048	0.059	0.069	0.248
	0	0.3	20	100	0.042	0.050	0.041	0.051	0.062	0.074	0.075	0.134	0.068	0.004
			20	200	0.042	0.054	0.047	0.053	0.075	0.085	0.058	0.083	0.059	0.002
			40	100	0.042	0.050	0.037	0.041	0.067	0.079	0.080	0.154	0.048	0.001
			40	200	0.042	0.050	0.036	0.044	0.077	0.088	0.068	0.088	0.045	0.000
	0	-0.3	20	100	0.043	0.051	0.029	0.036	0.043	0.051	0.067	0.080	0.070	0.170
			20	200	0.048	0.058	0.030	0.038	0.049	0.059	0.071	0.060	0.063	0.207
			40	100	0.027	0.032	0.014	0.018	0.028	0.033	0.056	0.069	0.055	0.210
			40	200	0.037	0.042	0.023	0.027	0.037	0.042	0.051	0.052	0.046	0.278
Sharp	0	0	20	100	0.058	0.071	0.056	0.065	0.056	0.065	0.094	0.142	0.091	0.055
			20	200	0.048	0.057	0.044	0.052	0.056	0.068	0.069	0.081	0.078	0.055
			40	100	0.042	0.048	0.029	0.031	0.039	0.043	0.078	0.117	0.099	0.054
			40	200	0.039	0.047	0.035	0.041	0.050	0.060	0.068	0.070	0.082	0.060
	0.3	0	20	100	0.038	0.045	0.041	0.051	0.044	0.052	0.056	0.114	0.088	0.052
			20	200	0.044	0.052	0.050	0.058	0.067	0.074	0.060	0.083	0.068	0.040
			40	100	0.029	0.032	0.034	0.037	0.028	0.033	0.047	0.113	0.084	0.044
			40	200	0.037	0.046	0.037	0.043	0.056	0.066	0.057	0.074	0.073	0.030
	-0.3	0	20	100	0.026	0.034	0.035	0.040	0.024	0.029	0.043	0.077	0.082	0.190
			20	200	0.044	0.051	0.046	0.056	0.045	0.052	0.057	0.081	0.076	0.206
			40	100	0.024	0.030	0.026	0.029	0.023	0.027	0.032	0.077	0.086	0.269
			40	200	0.038	0.044	0.040	0.046	0.039	0.045	0.052	0.079	0.069	0.259
	0	0.3	20	100	0.043	0.050	0.048	0.057	0.056	0.067	0.080	0.124	0.076	0.005
			20	200	0.043	0.052	0.040	0.049	0.066	0.078	0.062	0.073	0.057	0.004
			40	100	0.045	0.052	0.037	0.042	0.052	0.061	0.082	0.156	0.065	0.000
			40	200	0.034	0.039	0.036	0.041	0.070	0.078	0.060	0.082	0.045	0.001
	0	-0.3	20	100	0.040	0.047	0.025	0.032	0.036	0.042	0.066	0.076	0.071	0.179
			20	200	0.038	0.047	0.029	0.040	0.041	0.051	0.057	0.056	0.064	0.206
			40	100	0.026	0.031	0.016	0.018	0.024	0.027	0.056	0.074	0.069	0.255
			40	200	0.035	0.042	0.023	0.031	0.037	0.044	0.056	0.049	0.055	0.285
Smooth	0	0	20	100	0.045	0.054	0.039	0.045	0.048	0.056	0.079	0.113	0.011	0.008
			20	200	0.052	0.065	0.042	0.052	0.060	0.074	0.073	0.075	0.000	0.000
			40	100	0.040	0.046	0.027	0.029	0.040	0.046	0.084	0.114	0.107	0.080
			40	200	0.040	0.046	0.031	0.036	0.050	0.059	0.060	0.066	0.000	0.000

	0.3	0	20	100	0.034	0.040	0.033	0.040	0.039	0.051	0.051	0.091	0.023	0.017
			20	200	0.042	0.051	0.047	0.057	0.058	0.070	0.062	0.078	0.000	0.000
			40	100	0.026	0.030	0.029	0.032	0.032	0.035	0.040	0.108	0.140	0.120
			40	200	0.034	0.040	0.031	0.040	0.048	0.055	0.047	0.072	0.000	0.000
	-0.3	0	20	100	0.031	0.041	0.026	0.030	0.026	0.032	0.050	0.061	0.000	0.000
			20	200	0.046	0.053	0.047	0.057	0.047	0.054	0.063	0.070	0.000	0.000
			40	100	0.022	0.024	0.017	0.020	0.019	0.022	0.032	0.055	0.001	0.002
			40	200	0.038	0.046	0.031	0.034	0.038	0.045	0.049	0.062	0.000	0.000
	0	0.3	20	100	0.053	0.063	0.044	0.054	0.067	0.079	0.087	0.127	0.040	0.004
			20	200	0.040	0.051	0.045	0.054	0.079	0.091	0.064	0.083	0.000	0.000
			40	100	0.040	0.045	0.036	0.039	0.061	0.067	0.083	0.150	0.187	0.040
			40	200	0.040	0.048	0.032	0.038	0.072	0.083	0.064	0.073	0.000	0.000
	0	-0.3	20	100	0.038	0.046	0.027	0.032	0.035	0.042	0.064	0.069	0.000	0.001
			20	200	0.041	0.051	0.033	0.040	0.042	0.051	0.062	0.060	0.000	0.000
			40	100	0.027	0.033	0.017	0.019	0.027	0.032	0.053	0.064	0.001	0.003
			40	200	0.031	0.041	0.026	0.029	0.032	0.040	0.052	0.052	0.000	0.000
Mixed	0	0	20	100	0.051	0.059	0.042	0.055	0.059	0.068	0.095	0.119	0.000	0.000
			20	200	0.049	0.061	0.044	0.053	0.058	0.071	0.075	0.080	0.001	0.001
			40	100	0.052	0.056	0.036	0.042	0.053	0.061	0.089	0.127	0.000	0.000
			40	200	0.046	0.053	0.025	0.028	0.055	0.064	0.065	0.065	0.000	0.000
	0.3	0	20	100	0.038	0.044	0.042	0.049	0.043	0.050	0.052	0.102	0.002	0.000
			20	200	0.037	0.047	0.041	0.050	0.052	0.063	0.052	0.076	0.006	0.004
			40	100	0.029	0.033	0.032	0.038	0.030	0.036	0.045	0.119	0.000	0.000
			40	200	0.035	0.040	0.034	0.040	0.055	0.065	0.048	0.078	0.001	0.001
	-0.3	0	20	100	0.029	0.034	0.030	0.035	0.030	0.033	0.041	0.073	0.000	0.001
			20	200	0.038	0.046	0.035	0.042	0.038	0.045	0.050	0.060	0.000	0.003
			40	100	0.019	0.023	0.018	0.020	0.017	0.020	0.029	0.064	0.000	0.000
			40	200	0.036	0.041	0.035	0.038	0.037	0.042	0.052	0.067	0.000	0.000
	0	0.3	20	100	0.040	0.052	0.045	0.050	0.057	0.068	0.073	0.119	0.001	0.000
			20	200	0.041	0.045	0.036	0.046	0.069	0.081	0.055	0.074	0.004	0.001
			40	100	0.035	0.040	0.028	0.033	0.049	0.057	0.073	0.130	0.000	0.000
			40	200	0.035	0.038	0.026	0.031	0.061	0.070	0.056	0.066	0.001	0.000
	0	-0.3	20	100	0.028	0.035	0.022	0.029	0.028	0.036	0.052	0.068	0.000	0.000
			20	200	0.035	0.047	0.032	0.039	0.037	0.048	0.052	0.055	0.000	0.002
			40	100	0.023	0.026	0.011	0.013	0.021	0.024	0.044	0.060	0.000	0.000
			40	200	0.024	0.032	0.016	0.020	0.026	0.034	0.037	0.039	0.000	0.000

Notes: See Table 3.2.

Table 7.20. Power properties of panel statistics with $h = 5$

B. Type	θ	φ	N	T	$P(1)$	$P_M(1)$	$P(2)$	$P_M(2)$	$P(\hat{m}_i)$	$P_M(\hat{m}_i)$	$Z_\tau(1)$	$Z_\tau(2)$	Z_τ	Z_ϕ
No break	0	0	20	100	0.573	0.602	0.226	0.256	0.535	0.568	0.770	0.474	0.954	0.937
			20	200	1.000	1.000	0.955	0.965	1.000	1.000	1.000	0.995	1.000	1.000
			40	100	0.812	0.831	0.313	0.340	0.764	0.784	0.951	0.702	1.000	0.999
			40	200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.3	0	20	100	0.261	0.293	0.109	0.126	0.258	0.286	0.408	0.260	0.904	0.904
			20	200	0.993	0.993	0.748	0.772	0.992	0.993	0.999	0.901	1.000	1.000
			40	100	0.416	0.440	0.138	0.154	0.388	0.413	0.623	0.394	0.994	0.993
			40	200	1.000	1.000	0.958	0.961	1.000	1.000	1.000	0.996	1.000	1.000
	-0.3	0	20	100	0.265	0.298	0.099	0.117	0.245	0.274	0.403	0.234	0.556	0.789
			20	200	0.995	0.996	0.797	0.824	0.994	0.995	0.999	0.925	0.999	1.000
			40	100	0.424	0.453	0.116	0.127	0.388	0.415	0.616	0.316	0.879	0.978
			40	200	1.000	1.000	0.971	0.973	1.000	1.000	1.000	0.998	1.000	1.000
	0	0.3	20	100	0.513	0.546	0.211	0.238	0.509	0.543	0.724	0.461	0.983	0.807
			20	200	1.000	1.000	0.937	0.950	1.000	1.000	1.000	0.986	1.000	1.000
			40	100	0.779	0.803	0.295	0.316	0.751	0.773	0.938	0.697	0.999	0.966
			40	200	1.000	1.000	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000
	0	-0.3	20	100	0.514	0.553	0.187	0.208	0.489	0.528	0.730	0.408	0.847	0.967
			20	200	1.000	1.000	0.955	0.965	1.000	1.000	1.000	0.995	1.000	1.000
			40	100	0.780	0.801	0.249	0.272	0.727	0.750	0.944	0.603	0.970	0.997
			40	200	1.000	1.000	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000
Sharp	0	0	20	100	0.532	0.574	0.227	0.254	0.493	0.535	0.739	0.457	0.979	0.970
			20	200	0.999	0.999	0.938	0.949	0.998	0.999	1.000	0.992	1.000	1.000
			40	100	0.798	0.819	0.319	0.342	0.734	0.753	0.953	0.680	1.000	1.000
			40	200	1.000	1.000	0.998	0.998	1.000	1.000	1.000	1.000	1.000	1.000
	0.3	0	20	100	0.269	0.300	0.122	0.136	0.238	0.263	0.402	0.261	0.935	0.935
			20	200	0.986	0.989	0.713	0.742	0.985	0.988	0.997	0.885	1.000	1.000
			40	100	0.411	0.440	0.133	0.150	0.353	0.381	0.606	0.375	0.999	0.999
			40	200	1.000	1.000	0.951	0.959	1.000	1.000	1.000	0.996	1.000	1.000
	-0.3	0	20	100	0.254	0.284	0.091	0.103	0.223	0.249	0.377	0.213	0.735	0.915
			20	200	0.936	0.944	0.655	0.685	0.934	0.942	0.978	0.829	1.000	1.000
			40	100	0.394	0.421	0.101	0.114	0.358	0.383	0.586	0.319	0.967	0.999
			40	200	1.000	1.000	0.960	0.965	0.999	0.999	1.000	0.996	1.000	1.000
	0	0.3	20	100	0.482	0.517	0.199	0.232	0.443	0.473	0.709	0.447	0.989	0.865
			20	200	1.000	1.000	0.903	0.919	0.998	0.998	1.000	0.979	1.000	1.000
			40	100	0.741	0.763	0.295	0.317	0.678	0.704	0.919	0.674	1.000	0.988
			40	200	1.000	1.000	0.998	0.998	1.000	1.000	1.000	1.000	1.000	1.000
	0	-0.3	20	100	0.467	0.504	0.165	0.189	0.422	0.456	0.688	0.370	0.920	0.981
			20	200	0.991	0.993	0.845	0.865	0.991	0.992	1.000	0.957	1.000	1.000
			40	100	0.735	0.755	0.223	0.246	0.665	0.689	0.921	0.572	0.996	1.000
			40	200	1.000	1.000	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000
Smooth	0	0	20	100	0.540	0.576	0.201	0.233	0.513	0.546	0.760	0.441	0.002	0.002
			20	200	1.000	1.000	0.947	0.955	1.000	1.000	1.000	0.992	0.000	0.004
			40	100	0.808	0.827	0.287	0.307	0.759	0.781	0.958	0.672	0.005	0.004
			40	200	1.000	1.000	0.999	0.999	1.000	1.000	1.000	1.000	0.000	0.000

	0.3	0	20	100	0.267	0.303	0.103	0.122	0.246	0.272	0.415	0.262	0.001	0.002
			20	200	0.993	0.995	0.762	0.788	0.993	0.995	0.999	0.908	0.005	0.009
			40	100	0.421	0.449	0.119	0.137	0.382	0.405	0.639	0.365	0.001	0.002
			40	200	1.000	1.000	0.953	0.960	1.000	1.000	1.000	0.997	0.000	0.000
	-0.3	0	20	100	0.261	0.291	0.082	0.097	0.240	0.271	0.392	0.188	0.000	0.000
			20	200	0.996	0.996	0.788	0.815	0.995	0.995	0.999	0.927	0.000	0.000
			40	100	0.420	0.450	0.091	0.103	0.388	0.416	0.610	0.272	0.000	0.000
			40	200	1.000	1.000	0.973	0.977	1.000	1.000	1.000	0.997	0.000	0.000
	0	0.3	20	100	0.508	0.541	0.205	0.228	0.495	0.532	0.725	0.441	0.007	0.004
			20	200	1.000	1.000	0.928	0.943	1.000	1.000	1.000	0.989	0.041	0.016
			40	100	0.771	0.787	0.288	0.314	0.733	0.754	0.935	0.674	0.004	0.004
			40	200	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	0.005	0.001
	0	-0.3	20	100	0.502	0.542	0.146	0.169	0.470	0.508	0.724	0.360	0.000	0.001
			20	200	1.000	1.000	0.956	0.964	1.000	1.000	1.000	0.993	0.000	0.002
			40	100	0.760	0.784	0.198	0.222	0.708	0.729	0.942	0.539	0.000	0.000
			40	200	1.000	1.000	0.998	0.999	1.000	1.000	1.000	1.000	0.000	0.000
Mixed	0	0	20	100	0.494	0.527	0.191	0.217	0.465	0.495	0.709	0.434	0.007	0.052
			20	200	1.000	1.000	0.938	0.950	1.000	1.000	1.000	0.993	0.869	0.950
			40	100	0.750	0.774	0.275	0.298	0.698	0.723	0.929	0.652	0.004	0.067
			40	200	1.000	1.000	0.998	0.999	1.000	1.000	1.000	1.000	0.996	1.000
	0.3	0	20	100	0.253	0.284	0.102	0.122	0.237	0.267	0.383	0.262	0.027	0.058
			20	200	0.988	0.990	0.724	0.756	0.989	0.991	0.999	0.894	0.903	0.959
			40	100	0.392	0.419	0.129	0.145	0.366	0.391	0.590	0.367	0.034	0.077
			40	200	1.000	1.000	0.947	0.956	1.000	1.000	1.000	0.995	0.995	0.999
	-0.3	0	20	100	0.188	0.214	0.063	0.073	0.174	0.198	0.308	0.180	0.000	0.013
			20	200	0.982	0.985	0.694	0.723	0.980	0.983	0.996	0.868	0.065	0.518
			40	100	0.289	0.315	0.074	0.088	0.265	0.292	0.484	0.263	0.001	0.020
			40	200	1.000	1.000	0.929	0.942	0.999	0.999	1.000	0.993	0.124	0.890
	0	0.3	20	100	0.433	0.471	0.168	0.192	0.423	0.457	0.654	0.399	0.070	0.017
			20	200	0.999	0.999	0.907	0.920	0.999	0.999	1.000	0.980	0.996	0.949
			40	100	0.692	0.714	0.233	0.254	0.654	0.676	0.898	0.607	0.094	0.011
			40	200	1.000	1.000	0.997	0.997	1.000	1.000	1.000	1.000	1.000	1.000
	0	-0.3	20	100	0.367	0.404	0.113	0.128	0.345	0.379	0.587	0.304	0.000	0.024
			20	200	0.999	1.000	0.911	0.929	0.997	0.997	1.000	0.984	0.341	0.891
			40	100	0.603	0.630	0.160	0.172	0.557	0.582	0.841	0.464	0.000	0.058
			40	200	1.000	1.000	0.995	0.995	1.000	1.000	1.000	1.000	0.599	0.993

Notes: See Table 3.2.

Table 7.21. Size properties of panel statistics with $r_{max} = 2$

B. Type	θ	φ	N	T	$P(1)$	$P_M(1)$	$P(2)$	$P_M(2)$	$P(\hat{m}_i)$	$P_M(\hat{m}_i)$	$Z_\tau(1)$	$Z_\tau(2)$	Z_τ	Z_ϕ
No break	0	0	20	100	0.047	0.058	0.040	0.048	0.040	0.047	0.082	0.116	0.104	0.069
			20	200	0.052	0.062	0.046	0.054	0.048	0.060	0.073	0.078	0.077	0.060
			40	100	0.044	0.051	0.030	0.034	0.036	0.040	0.082	0.129	0.117	0.063
			40	200	0.041	0.046	0.034	0.041	0.040	0.044	0.060	0.075	0.085	0.058
	0.3	0	20	100	0.043	0.049	0.041	0.052	0.043	0.051	0.060	0.116	0.107	0.072
			20	200	0.048	0.057	0.042	0.052	0.048	0.057	0.067	0.081	0.082	0.048
			40	100	0.034	0.039	0.031	0.035	0.027	0.034	0.053	0.126	0.102	0.057
			40	200	0.037	0.043	0.036	0.041	0.042	0.050	0.054	0.082	0.078	0.034
	-0.3	0	20	100	0.033	0.041	0.026	0.032	0.028	0.034	0.045	0.064	0.077	0.171
			20	200	0.046	0.057	0.040	0.053	0.035	0.042	0.068	0.069	0.078	0.198
			40	100	0.025	0.029	0.017	0.020	0.018	0.022	0.039	0.068	0.096	0.241
			40	200	0.036	0.043	0.034	0.040	0.026	0.032	0.052	0.070	0.070	0.260
	0	0.3	20	100	0.054	0.064	0.056	0.063	0.064	0.073	0.097	0.147	0.078	0.005
			20	200	0.047	0.056	0.053	0.060	0.066	0.080	0.070	0.090	0.066	0.004
			40	100	0.050	0.058	0.039	0.046	0.059	0.069	0.099	0.160	0.073	0.001
			40	200	0.046	0.052	0.038	0.045	0.066	0.077	0.072	0.091	0.055	0.001
	0	-0.3	20	100	0.039	0.046	0.029	0.033	0.031	0.035	0.060	0.078	0.071	0.182
			20	200	0.047	0.056	0.039	0.046	0.037	0.045	0.066	0.065	0.067	0.212
			40	100	0.030	0.035	0.019	0.023	0.022	0.026	0.064	0.084	0.063	0.229
			40	200	0.034	0.040	0.026	0.029	0.027	0.029	0.048	0.054	0.051	0.285
Sharp	0	0	20	100	0.054	0.066	0.039	0.045	0.048	0.054	0.091	0.117	0.103	0.069
			20	200	0.047	0.057	0.038	0.049	0.042	0.050	0.072	0.074	0.079	0.060
			40	100	0.037	0.042	0.033	0.040	0.035	0.040	0.090	0.133	0.107	0.055
			40	200	0.048	0.053	0.034	0.039	0.041	0.047	0.072	0.071	0.091	0.067
	0.3	0	20	100	0.034	0.042	0.038	0.045	0.039	0.049	0.060	0.107	0.093	0.054
			20	200	0.042	0.051	0.048	0.056	0.050	0.060	0.058	0.082	0.076	0.046
			40	100	0.027	0.033	0.027	0.030	0.032	0.035	0.044	0.120	0.109	0.057
			40	200	0.033	0.039	0.036	0.042	0.047	0.055	0.052	0.081	0.073	0.036
	-0.3	0	20	100	0.030	0.034	0.025	0.031	0.024	0.028	0.045	0.074	0.087	0.199
			20	200	0.034	0.042	0.035	0.042	0.027	0.033	0.054	0.062	0.076	0.202
			40	100	0.018	0.020	0.015	0.019	0.011	0.013	0.035	0.066	0.093	0.264
			40	200	0.027	0.032	0.029	0.033	0.026	0.028	0.043	0.064	0.074	0.276
	0	0.3	20	100	0.034	0.043	0.028	0.034	0.034	0.043	0.058	0.087	0.086	0.004
			20	200	0.037	0.042	0.029	0.038	0.045	0.056	0.055	0.056	0.067	0.003
			40	100	0.022	0.025	0.015	0.019	0.036	0.042	0.046	0.088	0.084	0.000
			40	200	0.027	0.030	0.022	0.025	0.039	0.046	0.046	0.047	0.054	0.000
	0	-0.3	20	100	0.022	0.028	0.018	0.021	0.020	0.025	0.043	0.050	0.068	0.186
			20	200	0.036	0.042	0.021	0.026	0.028	0.033	0.048	0.041	0.070	0.216
			40	100	0.013	0.016	0.007	0.009	0.011	0.013	0.038	0.036	0.074	0.265
			40	200	0.017	0.019	0.009	0.012	0.013	0.016	0.031	0.028	0.056	0.311
Smooth	0	0	20	100	0.048	0.059	0.035	0.039	0.039	0.050	0.086	0.105	0.000	0.000
			20	200	0.057	0.067	0.048	0.058	0.051	0.060	0.076	0.083	0.000	0.000
			40	100	0.048	0.054	0.032	0.036	0.037	0.042	0.087	0.125	0.000	0.000
			40	200	0.044	0.052	0.033	0.040	0.044	0.052	0.074	0.074	0.000	0.000

	0.3	0	20	100	0.036	0.046	0.048	0.056	0.040	0.048	0.055	0.104	0.000	0.000
			20	200	0.041	0.052	0.047	0.057	0.045	0.055	0.060	0.084	0.000	0.000
			40	100	0.034	0.039	0.030	0.035	0.028	0.033	0.053	0.110	0.000	0.000
			40	200	0.036	0.041	0.038	0.046	0.038	0.044	0.052	0.073	0.000	0.000
	-0.3	0	20	100	0.035	0.042	0.027	0.033	0.028	0.033	0.045	0.066	0.000	0.000
			20	200	0.043	0.050	0.039	0.047	0.033	0.039	0.056	0.069	0.000	0.000
			40	100	0.019	0.022	0.013	0.015	0.015	0.017	0.034	0.049	0.000	0.000
			40	200	0.041	0.048	0.039	0.047	0.029	0.033	0.049	0.074	0.000	0.000
	0	0.3	20	100	0.055	0.064	0.049	0.060	0.067	0.080	0.089	0.135	0.000	0.000
			20	200	0.056	0.068	0.043	0.052	0.067	0.077	0.080	0.089	0.000	0.000
			40	100	0.051	0.057	0.044	0.050	0.053	0.059	0.101	0.160	0.000	0.000
			40	200	0.044	0.050	0.043	0.050	0.057	0.068	0.071	0.105	0.000	0.000
	0	-0.3	20	100	0.040	0.044	0.026	0.031	0.034	0.038	0.058	0.065	0.000	0.000
			20	200	0.045	0.053	0.033	0.038	0.033	0.041	0.061	0.054	0.000	0.000
			40	100	0.029	0.033	0.012	0.015	0.017	0.021	0.055	0.054	0.000	0.000
			40	200	0.043	0.050	0.024	0.029	0.033	0.037	0.063	0.055	0.000	0.000
Mixed	0	0	20	100	0.051	0.060	0.049	0.057	0.043	0.050	0.086	0.120	0.000	0.000
			20	200	0.048	0.056	0.041	0.050	0.044	0.053	0.074	0.079	0.000	0.000
			40	100	0.046	0.053	0.031	0.036	0.040	0.047	0.092	0.121	0.000	0.000
			40	200	0.047	0.053	0.034	0.039	0.041	0.045	0.065	0.068	0.000	0.000
	0.3	0	20	100	0.034	0.039	0.042	0.050	0.037	0.045	0.049	0.109	0.002	0.001
			20	200	0.045	0.055	0.051	0.063	0.053	0.064	0.061	0.081	0.004	0.003
			40	100	0.023	0.027	0.029	0.036	0.025	0.029	0.045	0.125	0.000	0.000
			40	200	0.032	0.040	0.035	0.039	0.049	0.055	0.051	0.079	0.002	0.001
	-0.3	0	20	100	0.028	0.034	0.023	0.029	0.019	0.022	0.041	0.073	0.000	0.001
			20	200	0.045	0.050	0.037	0.046	0.035	0.040	0.057	0.065	0.000	0.002
			40	100	0.019	0.022	0.018	0.021	0.013	0.016	0.031	0.073	0.000	0.000
			40	200	0.027	0.035	0.038	0.043	0.021	0.025	0.039	0.072	0.000	0.000
	0	0.3	20	100	0.051	0.057	0.037	0.048	0.056	0.066	0.079	0.109	0.002	0.000
			20	200	0.048	0.059	0.040	0.046	0.059	0.070	0.070	0.068	0.004	0.000
			40	100	0.027	0.032	0.029	0.033	0.035	0.041	0.064	0.114	0.000	0.000
			40	200	0.031	0.036	0.032	0.038	0.043	0.048	0.055	0.070	0.000	0.000
	0	-0.3	20	100	0.028	0.036	0.022	0.025	0.023	0.030	0.048	0.057	0.000	0.001
			20	200	0.037	0.043	0.031	0.040	0.032	0.038	0.053	0.059	0.000	0.003
			40	100	0.020	0.024	0.013	0.014	0.014	0.016	0.044	0.055	0.000	0.000
			40	200	0.028	0.031	0.022	0.027	0.019	0.022	0.044	0.049	0.000	0.001

Notes: See Table 3.2.

Table 7.22. Power properties of panel statistics with $r_{max} = 2$

B. Type	θ	φ	N	T	$P(1)$	$P_M(1)$	$P(2)$	$P_M(2)$	$P(\hat{m}_i)$	$P_M(\hat{m}_i)$	$Z_\tau(1)$	$Z_\tau(2)$	Z_τ	Z_ϕ
No break	0	0	20	100	0.565	0.599	0.219	0.245	0.460	0.494	0.767	0.473	0.956	0.943
			20	200	1.000	1.000	0.958	0.963	0.971	0.974	1.000	0.993	1.000	1.000
			40	100	0.814	0.831	0.315	0.334	0.650	0.674	0.963	0.693	0.999	0.999
			40	200	1.000	1.000	1.000	1.000	0.992	0.993	1.000	1.000	1.000	1.000
	0.3	0	20	100	0.274	0.301	0.101	0.119	0.230	0.253	0.425	0.259	0.894	0.887
			20	200	0.989	0.992	0.762	0.789	0.934	0.945	0.998	0.910	1.000	1.000
			40	100	0.427	0.457	0.125	0.142	0.332	0.352	0.637	0.390	0.994	0.993
			40	200	1.000	1.000	0.953	0.959	0.981	0.983	1.000	0.995	1.000	1.000
	-0.3	0	20	100	0.253	0.283	0.095	0.112	0.208	0.234	0.383	0.221	0.608	0.819
			20	200	0.994	0.994	0.793	0.820	0.873	0.876	0.998	0.925	0.998	0.999
			40	100	0.397	0.427	0.109	0.122	0.305	0.328	0.585	0.312	0.854	0.975
			40	200	1.000	1.000	0.973	0.977	0.925	0.927	1.000	0.998	1.000	1.000
	0	0.3	20	100	0.534	0.571	0.217	0.242	0.447	0.487	0.751	0.484	0.974	0.789
			20	200	0.999	1.000	0.932	0.946	0.981	0.984	1.000	0.988	1.000	1.000
			40	100	0.779	0.800	0.318	0.344	0.660	0.684	0.945	0.708	1.000	0.981
			40	200	1.000	1.000	0.998	0.999	0.999	0.999	1.000	1.000	1.000	1.000
	0	-0.3	20	100	0.470	0.510	0.164	0.187	0.374	0.403	0.699	0.388	0.801	0.937
			20	200	1.000	1.000	0.950	0.961	0.945	0.949	1.000	0.995	1.000	1.000
			40	100	0.760	0.783	0.238	0.266	0.611	0.635	0.932	0.600	0.972	0.998
			40	200	1.000	1.000	0.999	0.999	0.972	0.973	1.000	1.000	1.000	1.000
Sharp	0	0	20	100	0.385	0.418	0.150	0.171	0.331	0.359	0.598	0.367	0.973	0.972
			20	200	0.999	0.999	0.899	0.913	0.958	0.960	1.000	0.975	1.000	1.000
			40	100	0.585	0.612	0.209	0.229	0.491	0.514	0.819	0.558	1.000	1.000
			40	200	1.000	1.000	0.993	0.993	0.987	0.988	1.000	1.000	1.000	1.000
	0.3	0	20	100	0.217	0.243	0.095	0.110	0.185	0.206	0.337	0.238	0.909	0.911
			20	200	0.967	0.978	0.677	0.716	0.904	0.916	0.995	0.862	1.000	1.000
			40	100	0.315	0.343	0.115	0.130	0.266	0.287	0.517	0.356	0.998	0.998
			40	200	1.000	1.000	0.904	0.917	0.973	0.976	1.000	0.990	1.000	1.000
	-0.3	0	20	100	0.124	0.145	0.060	0.070	0.094	0.111	0.212	0.162	0.767	0.925
			20	200	0.849	0.873	0.520	0.553	0.722	0.744	0.949	0.723	0.999	1.000
			40	100	0.147	0.165	0.054	0.060	0.105	0.116	0.283	0.210	0.962	0.997
			40	200	0.982	0.985	0.774	0.797	0.869	0.874	0.998	0.943	1.000	1.000
	0	0.3	20	100	0.344	0.378	0.126	0.142	0.307	0.340	0.562	0.332	0.984	0.808
			20	200	0.999	0.999	0.868	0.884	0.972	0.978	1.000	0.968	1.000	1.000
			40	100	0.543	0.572	0.159	0.179	0.465	0.487	0.798	0.489	1.000	0.989
			40	200	1.000	1.000	0.990	0.991	0.994	0.995	1.000	1.000	1.000	1.000
	0	-0.3	20	100	0.219	0.243	0.073	0.085	0.169	0.189	0.387	0.214	0.853	0.967
			20	200	0.982	0.987	0.763	0.788	0.900	0.906	0.997	0.914	1.000	1.000
			40	100	0.317	0.343	0.085	0.092	0.245	0.264	0.566	0.287	0.997	1.000
			40	200	1.000	1.000	0.945	0.954	0.943	0.946	1.000	0.996	1.000	1.000
Smooth	0	0	20	100	0.562	0.600	0.199	0.226	0.454	0.484	0.772	0.454	0.000	0.000
			20	200	1.000	1.000	0.951	0.961	0.967	0.971	1.000	0.994	0.002	0.004
			40	100	0.820	0.835	0.300	0.326	0.638	0.663	0.958	0.680	0.000	0.000
			40	200	1.000	1.000	0.999	0.999	0.993	0.994	1.000	1.000	0.000	0.000

	0.3	0	20	100	0.254	0.289	0.096	0.115	0.197	0.221	0.397	0.243	0.000	0.000
			20	200	0.993	0.996	0.761	0.792	0.937	0.945	0.999	0.914	0.006	0.015
			40	100	0.425	0.452	0.124	0.138	0.307	0.334	0.634	0.359	0.000	0.000
			40	200	1.000	1.000	0.955	0.961	0.976	0.978	1.000	0.996	0.000	0.001
	-0.3	0	20	100	0.262	0.294	0.086	0.100	0.199	0.223	0.386	0.206	0.000	0.000
			20	200	0.992	0.994	0.784	0.807	0.881	0.891	0.999	0.916	0.000	0.000
			40	100	0.392	0.423	0.090	0.102	0.299	0.320	0.591	0.269	0.000	0.000
			40	200	1.000	1.000	0.963	0.971	0.910	0.913	1.000	0.996	0.000	0.000
	0	0.3	20	100	0.496	0.533	0.193	0.220	0.411	0.450	0.731	0.444	0.000	0.000
			20	200	1.000	1.000	0.939	0.947	0.984	0.986	1.000	0.991	0.042	0.011
			40	100	0.764	0.784	0.293	0.325	0.618	0.643	0.945	0.682	0.000	0.000
			40	200	1.000	1.000	0.999	0.999	0.998	0.998	1.000	1.000	0.008	0.001
	0	-0.3	20	100	0.501	0.539	0.167	0.189	0.394	0.425	0.728	0.358	0.000	0.000
			20	200	1.000	1.000	0.950	0.959	0.947	0.950	1.000	0.994	0.000	0.002
			40	100	0.751	0.776	0.203	0.222	0.589	0.610	0.934	0.542	0.000	0.000
			40	200	1.000	1.000	0.999	0.999	0.975	0.977	1.000	1.000	0.000	0.000
Mixed	0	0	20	100	0.490	0.527	0.197	0.224	0.402	0.435	0.696	0.432	0.004	0.038
			20	200	0.999	0.999	0.929	0.941	0.960	0.963	1.000	0.986	0.842	0.932
			40	100	0.734	0.761	0.268	0.294	0.594	0.620	0.921	0.654	0.005	0.051
			40	200	1.000	1.000	0.998	0.998	0.989	0.991	1.000	1.000	0.994	1.000
	0.3	0	20	100	0.251	0.279	0.091	0.107	0.200	0.227	0.388	0.246	0.025	0.052
			20	200	0.988	0.991	0.727	0.756	0.920	0.930	0.998	0.895	0.882	0.944
			40	100	0.366	0.393	0.132	0.145	0.287	0.308	0.577	0.372	0.040	0.085
			40	200	1.000	1.000	0.944	0.951	0.980	0.982	1.000	0.995	0.998	1.000
	-0.3	0	20	100	0.178	0.205	0.074	0.088	0.137	0.158	0.292	0.187	0.000	0.013
			20	200	0.962	0.969	0.668	0.706	0.841	0.850	0.993	0.856	0.065	0.520
			40	100	0.265	0.288	0.067	0.078	0.195	0.214	0.432	0.252	0.000	0.014
			40	200	1.000	1.000	0.912	0.922	0.907	0.909	1.000	0.988	0.132	0.871
	0	0.3	20	100	0.461	0.493	0.165	0.188	0.384	0.418	0.667	0.392	0.065	0.013
			20	200	1.000	1.000	0.909	0.922	0.978	0.981	1.000	0.982	0.996	0.949
			40	100	0.700	0.726	0.232	0.258	0.582	0.605	0.903	0.628	0.175	0.019
			40	200	1.000	1.000	0.995	0.996	0.997	0.998	1.000	1.000	1.000	1.000
	0	-0.3	20	100	0.356	0.399	0.119	0.137	0.284	0.315	0.570	0.308	0.000	0.051
			20	200	0.999	0.999	0.895	0.917	0.929	0.935	1.000	0.976	0.283	0.833
			40	100	0.550	0.576	0.142	0.154	0.422	0.445	0.812	0.422	0.000	0.072
			40	200	1.000	1.000	0.994	0.995	0.963	0.965	1.000	1.000	0.675	0.996

Notes: See Table 3.2.

Table 7.23. Size properties of panel statistics with $r_{max} = 3$

B. Type	θ	φ	N	T	$P(1)$	$P_M(1)$	$P(2)$	$P_M(2)$	$P(\hat{m}_i)$	$P_M(\hat{m}_i)$	$Z_\tau(1)$	$Z_\tau(2)$	Z_τ	Z_ϕ
No break	0	0	20	100	0.056	0.070	0.042	0.051	0.047	0.060	0.094	0.118	0.112	0.074
			20	200	0.055	0.063	0.046	0.056	0.048	0.056	0.075	0.085	0.084	0.066
			40	100	0.051	0.059	0.037	0.042	0.039	0.045	0.086	0.134	0.117	0.070
			40	200	0.055	0.060	0.037	0.041	0.046	0.052	0.074	0.082	0.095	0.068
	0.3	0	20	100	0.040	0.044	0.041	0.051	0.041	0.048	0.053	0.111	0.090	0.057
			20	200	0.044	0.051	0.042	0.050	0.049	0.059	0.054	0.082	0.076	0.042
			40	100	0.030	0.033	0.035	0.040	0.028	0.032	0.050	0.122	0.093	0.055
			40	200	0.034	0.041	0.043	0.050	0.038	0.043	0.052	0.085	0.068	0.032
	-0.3	0	20	100	0.031	0.039	0.031	0.039	0.026	0.032	0.047	0.074	0.082	0.179
			20	200	0.041	0.052	0.051	0.058	0.034	0.040	0.064	0.079	0.073	0.197
			40	100	0.025	0.028	0.025	0.029	0.020	0.021	0.031	0.077	0.094	0.254
			40	200	0.036	0.040	0.032	0.036	0.022	0.027	0.052	0.066	0.080	0.268
	0	0.3	20	100	0.053	0.065	0.048	0.058	0.057	0.067	0.089	0.142	0.080	0.004
			20	200	0.043	0.055	0.041	0.049	0.054	0.061	0.063	0.076	0.067	0.002
			40	100	0.046	0.055	0.046	0.052	0.057	0.065	0.094	0.163	0.066	0.001
			40	200	0.041	0.050	0.048	0.054	0.058	0.067	0.063	0.085	0.057	0.000
	0	-0.3	20	100	0.039	0.051	0.027	0.035	0.035	0.041	0.062	0.083	0.066	0.173
			20	200	0.045	0.054	0.036	0.043	0.030	0.035	0.065	0.064	0.054	0.213
			40	100	0.037	0.041	0.015	0.019	0.028	0.030	0.057	0.070	0.076	0.253
			40	200	0.044	0.049	0.026	0.031	0.032	0.036	0.061	0.052	0.050	0.279
Sharp	0	0	20	100	0.067	0.074	0.050	0.057	0.045	0.054	0.099	0.115	0.105	0.069
			20	200	0.058	0.065	0.052	0.061	0.051	0.062	0.084	0.094	0.076	0.055
			40	100	0.052	0.057	0.037	0.041	0.034	0.040	0.088	0.147	0.114	0.062
			40	200	0.048	0.054	0.034	0.042	0.035	0.042	0.069	0.077	0.080	0.060
	0.3	0	20	100	0.035	0.043	0.041	0.047	0.030	0.039	0.061	0.105	0.088	0.058
			20	200	0.047	0.055	0.051	0.062	0.054	0.065	0.060	0.090	0.075	0.045
			40	100	0.033	0.037	0.036	0.042	0.027	0.029	0.049	0.123	0.106	0.053
			40	200	0.033	0.040	0.043	0.052	0.040	0.046	0.055	0.091	0.083	0.040
	-0.3	0	20	100	0.033	0.041	0.027	0.036	0.022	0.028	0.047	0.079	0.083	0.188
			20	200	0.040	0.048	0.042	0.049	0.032	0.040	0.050	0.067	0.072	0.192
			40	100	0.026	0.030	0.021	0.024	0.015	0.019	0.034	0.075	0.085	0.270
			40	200	0.043	0.050	0.040	0.047	0.029	0.033	0.056	0.071	0.075	0.272
	0	0.3	20	100	0.053	0.065	0.043	0.052	0.060	0.070	0.086	0.138	0.085	0.005
			20	200	0.048	0.057	0.046	0.051	0.056	0.064	0.071	0.080	0.056	0.005
			40	100	0.045	0.050	0.035	0.041	0.041	0.047	0.094	0.167	0.089	0.001
			40	200	0.042	0.049	0.037	0.044	0.053	0.058	0.069	0.086	0.063	0.000
	0	-0.3	20	100	0.044	0.048	0.030	0.036	0.028	0.034	0.068	0.081	0.078	0.198
			20	200	0.039	0.044	0.031	0.037	0.029	0.035	0.058	0.055	0.066	0.206
			40	100	0.030	0.037	0.018	0.021	0.017	0.019	0.060	0.070	0.071	0.265
			40	200	0.032	0.039	0.027	0.032	0.020	0.024	0.050	0.055	0.064	0.287
Smooth	0	0	20	100	0.055	0.065	0.040	0.050	0.045	0.054	0.084	0.116	0.013	0.010
			20	200	0.043	0.053	0.040	0.050	0.039	0.048	0.069	0.075	0.000	0.000
			40	100	0.050	0.058	0.029	0.036	0.037	0.044	0.089	0.119	0.098	0.078
			40	200	0.038	0.045	0.032	0.037	0.035	0.039	0.065	0.070	0.000	0.000

	0.3	0	20	100	0.034	0.045	0.038	0.045	0.033	0.040	0.056	0.098	0.025	0.019
			20	200	0.038	0.047	0.040	0.052	0.048	0.057	0.052	0.079	0.000	0.000
			40	100	0.027	0.033	0.028	0.037	0.031	0.035	0.047	0.113	0.115	0.098
			40	200	0.036	0.042	0.037	0.043	0.037	0.045	0.056	0.082	0.000	0.000
	-0.3	0	20	100	0.039	0.045	0.025	0.031	0.028	0.033	0.047	0.063	0.001	0.002
			20	200	0.042	0.050	0.038	0.046	0.031	0.037	0.053	0.067	0.000	0.000
			40	100	0.023	0.026	0.016	0.019	0.015	0.020	0.036	0.063	0.002	0.004
			40	200	0.033	0.036	0.030	0.036	0.026	0.029	0.046	0.064	0.000	0.000
	0	0.3	20	100	0.051	0.061	0.050	0.060	0.061	0.071	0.085	0.132	0.040	0.005
			20	200	0.051	0.063	0.045	0.057	0.062	0.073	0.076	0.081	0.000	0.000
			40	100	0.053	0.058	0.029	0.035	0.044	0.050	0.098	0.151	0.195	0.049
			40	200	0.043	0.049	0.042	0.048	0.055	0.067	0.072	0.088	0.000	0.000
	0	-0.3	20	100	0.041	0.048	0.022	0.028	0.029	0.034	0.057	0.072	0.001	0.001
			20	200	0.045	0.055	0.037	0.044	0.031	0.040	0.061	0.064	0.000	0.000
			40	100	0.026	0.031	0.013	0.015	0.014	0.019	0.051	0.056	0.003	0.005
			40	200	0.042	0.047	0.027	0.032	0.032	0.037	0.060	0.048	0.000	0.000
Mixed	0	0	20	100	0.047	0.058	0.046	0.055	0.041	0.051	0.081	0.118	0.000	0.000
			20	200	0.049	0.057	0.040	0.049	0.042	0.056	0.074	0.079	0.000	0.000
			40	100	0.045	0.051	0.035	0.040	0.038	0.044	0.087	0.133	0.000	0.000
			40	200	0.038	0.045	0.031	0.035	0.037	0.042	0.067	0.069	0.000	0.000
	0.3	0	20	100	0.039	0.047	0.046	0.053	0.034	0.044	0.059	0.113	0.002	0.001
			20	200	0.046	0.056	0.042	0.054	0.054	0.066	0.058	0.084	0.005	0.003
			40	100	0.030	0.037	0.040	0.045	0.031	0.037	0.056	0.124	0.000	0.000
			40	200	0.036	0.041	0.038	0.042	0.041	0.046	0.054	0.081	0.000	0.000
	-0.3	0	20	100	0.027	0.034	0.022	0.026	0.021	0.026	0.044	0.071	0.000	0.000
			20	200	0.035	0.043	0.037	0.047	0.027	0.031	0.048	0.066	0.000	0.001
			40	100	0.018	0.022	0.019	0.020	0.017	0.019	0.032	0.072	0.000	0.000
			40	200	0.036	0.045	0.035	0.041	0.030	0.034	0.048	0.072	0.000	0.000
	0	0.3	20	100	0.043	0.052	0.041	0.050	0.052	0.063	0.073	0.120	0.001	0.000
			20	200	0.052	0.059	0.035	0.041	0.059	0.069	0.075	0.076	0.005	0.000
			40	100	0.044	0.050	0.040	0.046	0.049	0.056	0.086	0.156	0.000	0.000
			40	200	0.040	0.044	0.036	0.042	0.048	0.055	0.058	0.080	0.000	0.000
	0	-0.3	20	100	0.031	0.038	0.021	0.025	0.023	0.027	0.049	0.066	0.000	0.001
			20	200	0.031	0.040	0.028	0.033	0.027	0.034	0.054	0.054	0.000	0.003
			40	100	0.029	0.033	0.015	0.017	0.021	0.025	0.058	0.060	0.000	0.000
			40	200	0.027	0.033	0.019	0.024	0.018	0.022	0.043	0.044	0.000	0.000

Notes: See Table 3.2.

Table 7.24. Power properties of panel statistics with $r_{max} = 3$

B. Type	θ	φ	N	T	$P(1)$	$P_M(1)$	$P(2)$	$P_M(2)$	$P(\hat{m}_i)$	$P_M(\hat{m}_i)$	$Z_\tau(1)$	$Z_\tau(2)$	Z_τ	Z_ϕ
No break	0	0	20	100	0.560	0.601	0.218	0.243	0.460	0.492	0.777	0.468	0.954	0.935
			20	200	1.000	1.000	0.958	0.966	0.969	0.971	1.000	0.994	1.000	1.000
			40	100	0.823	0.844	0.333	0.360	0.670	0.690	0.964	0.713	1.000	0.998
			40	200	1.000	1.000	1.000	1.000	0.988	0.989	1.000	1.000	1.000	1.000
	0.3	0	20	100	0.264	0.294	0.112	0.131	0.216	0.246	0.418	0.263	0.908	0.890
			20	200	0.993	0.995	0.769	0.798	0.936	0.945	1.000	0.917	1.000	1.000
			40	100	0.421	0.452	0.145	0.156	0.325	0.351	0.635	0.386	0.995	0.994
			40	200	1.000	1.000	0.955	0.963	0.981	0.982	1.000	0.997	1.000	1.000
	-0.3	0	20	100	0.253	0.284	0.086	0.103	0.196	0.219	0.380	0.218	0.639	0.831
			20	200	0.992	0.996	0.785	0.807	0.886	0.894	0.999	0.922	0.998	1.000
			40	100	0.405	0.436	0.110	0.125	0.306	0.332	0.603	0.323	0.865	0.977
			40	200	1.000	1.000	0.971	0.977	0.927	0.929	1.000	0.997	1.000	1.000
	0	0.3	20	100	0.519	0.555	0.218	0.247	0.460	0.493	0.731	0.469	0.975	0.789
			20	200	1.000	1.000	0.936	0.948	0.984	0.986	1.000	0.988	1.000	1.000
			40	100	0.794	0.810	0.323	0.347	0.671	0.695	0.949	0.702	1.000	0.970
			40	200	1.000	1.000	0.998	0.999	0.998	0.998	1.000	1.000	1.000	1.000
	0	-0.3	20	100	0.508	0.544	0.172	0.196	0.413	0.439	0.714	0.384	0.823	0.951
			20	200	1.000	1.000	0.958	0.965	0.944	0.946	1.000	0.993	1.000	1.000
			40	100	0.758	0.776	0.237	0.259	0.604	0.624	0.935	0.599	0.981	0.998
			40	200	1.000	1.000	1.000	1.000	0.975	0.975	1.000	1.000	1.000	1.000
Sharp	0	0	20	100	0.527	0.567	0.213	0.243	0.404	0.432	0.735	0.466	0.984	0.979
			20	200	1.000	1.000	0.939	0.947	0.958	0.962	1.000	0.990	1.000	1.000
			40	100	0.799	0.820	0.310	0.333	0.612	0.634	0.954	0.683	1.000	1.000
			40	200	1.000	1.000	0.998	0.998	0.989	0.989	1.000	1.000	1.000	1.000
	0.3	0	20	100	0.273	0.303	0.110	0.135	0.205	0.227	0.406	0.270	0.950	0.937
			20	200	0.983	0.987	0.720	0.747	0.910	0.920	0.997	0.878	1.000	1.000
			40	100	0.430	0.458	0.137	0.151	0.292	0.314	0.632	0.390	0.999	0.999
			40	200	1.000	1.000	0.952	0.961	0.974	0.976	1.000	0.996	1.000	1.000
	-0.3	0	20	100	0.236	0.265	0.090	0.103	0.172	0.192	0.360	0.221	0.766	0.915
			20	200	0.923	0.931	0.629	0.657	0.792	0.807	0.974	0.807	1.000	1.000
			40	100	0.390	0.415	0.103	0.116	0.267	0.285	0.586	0.317	0.957	0.995
			40	200	1.000	1.000	0.957	0.964	0.907	0.909	1.000	0.994	1.000	1.000
	0	0.3	20	100	0.485	0.519	0.212	0.234	0.391	0.421	0.718	0.459	0.988	0.810
			20	200	0.999	0.999	0.902	0.919	0.977	0.981	1.000	0.980	1.000	1.000
			40	100	0.774	0.790	0.304	0.330	0.594	0.617	0.934	0.684	1.000	0.983
			40	200	1.000	1.000	0.998	0.999	0.996	0.996	1.000	1.000	1.000	1.000
	0	-0.3	20	100	0.478	0.512	0.163	0.182	0.363	0.390	0.689	0.372	0.901	0.977
			20	200	0.991	0.993	0.844	0.872	0.914	0.919	0.999	0.959	1.000	1.000
			40	100	0.713	0.738	0.224	0.249	0.518	0.538	0.907	0.552	0.997	1.000
			40	200	1.000	1.000	0.997	0.998	0.960	0.962	1.000	1.000	1.000	1.000
Smooth	0	0	20	100	0.553	0.589	0.206	0.233	0.437	0.468	0.761	0.442	0.002	0.003
			20	200	1.000	1.000	0.955	0.966	0.968	0.971	1.000	0.995	0.003	0.008
			40	100	0.819	0.837	0.308	0.330	0.650	0.671	0.961	0.671	0.003	0.003
			40	200	1.000	1.000	0.999	0.999	0.989	0.991	1.000	1.000	0.000	0.000

	0.3	0	20	100	0.266	0.296	0.093	0.113	0.206	0.230	0.414	0.252	0.002	0.001
			20	200	0.992	0.994	0.743	0.774	0.934	0.942	1.000	0.898	0.005	0.011
			40	100	0.437	0.462	0.119	0.135	0.300	0.328	0.632	0.367	0.002	0.002
			40	200	1.000	1.000	0.955	0.964	0.982	0.984	1.000	0.997	0.000	0.001
	-0.3	0	20	100	0.247	0.271	0.074	0.090	0.189	0.209	0.365	0.178	0.000	0.000
			20	200	0.992	0.994	0.778	0.806	0.878	0.885	0.999	0.918	0.000	0.001
			40	100	0.403	0.431	0.090	0.105	0.290	0.313	0.583	0.261	0.000	0.000
			40	200	1.000	1.000	0.968	0.972	0.924	0.926	1.000	0.996	0.000	0.000
	0	0.3	20	100	0.516	0.553	0.196	0.229	0.432	0.461	0.742	0.455	0.004	0.001
			20	200	1.000	1.000	0.940	0.950	0.981	0.984	1.000	0.990	0.049	0.019
			40	100	0.773	0.791	0.282	0.306	0.634	0.660	0.948	0.673	0.006	0.006
			40	200	1.000	1.000	0.999	0.999	0.997	0.998	1.000	1.000	0.010	0.002
	0	-0.3	20	100	0.487	0.524	0.145	0.164	0.375	0.410	0.696	0.343	0.000	0.000
			20	200	1.000	1.000	0.947	0.957	0.946	0.949	1.000	0.990	0.000	0.003
			40	100	0.745	0.770	0.201	0.220	0.576	0.598	0.929	0.532	0.000	0.000
			40	200	1.000	1.000	0.998	0.998	0.975	0.977	1.000	1.000	0.000	0.000
Mixed	0	0	20	100	0.498	0.532	0.192	0.214	0.403	0.437	0.711	0.440	0.006	0.034
			20	200	1.000	1.000	0.935	0.946	0.965	0.969	1.000	0.989	0.826	0.935
			40	100	0.769	0.789	0.297	0.324	0.610	0.633	0.940	0.672	0.008	0.071
			40	200	1.000	1.000	0.998	0.999	0.987	0.988	1.000	1.000	0.998	1.000
	0.3	0	20	100	0.257	0.289	0.109	0.125	0.204	0.227	0.399	0.255	0.028	0.057
			20	200	0.990	0.992	0.724	0.757	0.923	0.930	0.999	0.898	0.910	0.969
			40	100	0.386	0.416	0.131	0.149	0.288	0.314	0.598	0.376	0.024	0.079
			40	200	1.000	1.000	0.941	0.950	0.977	0.981	1.000	0.994	0.996	1.000
	-0.3	0	20	100	0.179	0.205	0.074	0.085	0.134	0.158	0.295	0.189	0.000	0.015
			20	200	0.976	0.982	0.679	0.706	0.848	0.859	0.996	0.860	0.065	0.547
			40	100	0.292	0.319	0.078	0.088	0.208	0.229	0.477	0.277	0.000	0.013
			40	200	1.000	1.000	0.912	0.921	0.913	0.916	1.000	0.985	0.097	0.835
	0	0.3	20	100	0.456	0.500	0.186	0.212	0.387	0.421	0.677	0.418	0.101	0.020
			20	200	1.000	1.000	0.914	0.927	0.985	0.987	1.000	0.980	0.998	0.948
			40	100	0.727	0.748	0.265	0.287	0.584	0.610	0.913	0.650	0.127	0.012
			40	200	1.000	1.000	0.997	0.998	0.999	0.999	1.000	1.000	1.000	1.000
	0	-0.3	20	100	0.380	0.406	0.119	0.137	0.296	0.325	0.568	0.298	0.000	0.033
			20	200	1.000	1.000	0.894	0.910	0.930	0.936	1.000	0.980	0.319	0.869
			40	100	0.591	0.618	0.159	0.178	0.451	0.473	0.842	0.470	0.000	0.056
			40	200	1.000	1.000	0.995	0.996	0.961	0.964	1.000	1.000	0.612	0.995

Notes: See Table 3.2.

Table 7.25. Size properties of panel statistics with $r_{max} = 4$

B. Type	θ	φ	N	T	$P(1)$	$P_M(1)$	$P(2)$	$P_M(2)$	$P(\hat{m}_i)$	$P_M(\hat{m}_i)$	$Z_\tau(1)$	$Z_\tau(2)$	Z_τ	Z_ϕ
No break	0	0	20	100	0.049	0.058	0.050	0.056	0.044	0.051	0.086	0.126	0.106	0.072
			20	200	0.048	0.062	0.044	0.053	0.045	0.054	0.076	0.081	0.086	0.066
			40	100	0.047	0.055	0.036	0.042	0.041	0.047	0.087	0.138	0.114	0.067
			40	200	0.051	0.054	0.041	0.046	0.040	0.045	0.075	0.079	0.088	0.066
	0.3	0	20	100	0.034	0.039	0.046	0.054	0.038	0.045	0.056	0.107	0.094	0.053
			20	200	0.041	0.049	0.048	0.057	0.049	0.061	0.055	0.078	0.080	0.048
			40	100	0.026	0.032	0.032	0.040	0.021	0.025	0.047	0.124	0.099	0.051
			40	200	0.035	0.042	0.042	0.047	0.045	0.051	0.056	0.087	0.071	0.033
	-0.3	0	20	100	0.030	0.036	0.032	0.040	0.021	0.026	0.041	0.085	0.090	0.180
			20	200	0.057	0.068	0.044	0.053	0.041	0.049	0.066	0.079	0.083	0.203
			40	100	0.022	0.026	0.021	0.025	0.018	0.020	0.035	0.067	0.089	0.266
			40	200	0.037	0.043	0.031	0.036	0.027	0.031	0.051	0.066	0.069	0.269
	0	0.3	20	100	0.048	0.058	0.053	0.060	0.061	0.077	0.082	0.130	0.073	0.003
			20	200	0.060	0.068	0.051	0.060	0.069	0.081	0.077	0.095	0.069	0.004
			40	100	0.043	0.052	0.040	0.046	0.055	0.065	0.089	0.156	0.075	0.000
			40	200	0.052	0.059	0.042	0.047	0.068	0.078	0.081	0.085	0.055	0.000
	0	-0.3	20	100	0.045	0.053	0.031	0.036	0.033	0.038	0.071	0.075	0.069	0.166
			20	200	0.045	0.051	0.037	0.045	0.036	0.041	0.059	0.062	0.066	0.208
			40	100	0.034	0.039	0.020	0.024	0.020	0.025	0.064	0.082	0.067	0.233
			40	200	0.038	0.046	0.028	0.032	0.028	0.032	0.051	0.055	0.054	0.280
Sharp	0	0	20	100	0.067	0.077	0.048	0.057	0.056	0.063	0.108	0.123	0.107	0.067
			20	200	0.057	0.067	0.051	0.061	0.050	0.060	0.075	0.087	0.087	0.064
			40	100	0.048	0.056	0.033	0.039	0.030	0.034	0.089	0.142	0.113	0.062
			40	200	0.038	0.044	0.031	0.039	0.030	0.033	0.071	0.076	0.084	0.060
	0.3	0	20	100	0.036	0.043	0.043	0.053	0.034	0.041	0.056	0.116	0.100	0.062
			20	200	0.054	0.063	0.046	0.056	0.055	0.062	0.068	0.093	0.081	0.049
			40	100	0.032	0.038	0.035	0.040	0.025	0.030	0.050	0.123	0.094	0.047
			40	200	0.034	0.039	0.040	0.050	0.038	0.043	0.051	0.084	0.078	0.038
	-0.3	0	20	100	0.037	0.043	0.037	0.043	0.030	0.033	0.053	0.090	0.102	0.206
			20	200	0.045	0.052	0.040	0.049	0.034	0.042	0.062	0.076	0.080	0.209
			40	100	0.023	0.029	0.016	0.021	0.013	0.017	0.038	0.068	0.100	0.283
			40	200	0.038	0.043	0.031	0.040	0.024	0.029	0.056	0.069	0.074	0.276
	0	0.3	20	100	0.055	0.065	0.053	0.064	0.059	0.068	0.099	0.137	0.080	0.005
			20	200	0.048	0.057	0.043	0.053	0.063	0.075	0.077	0.083	0.068	0.005
			40	100	0.045	0.053	0.039	0.045	0.041	0.047	0.089	0.165	0.082	0.000
			40	200	0.046	0.055	0.040	0.044	0.052	0.059	0.075	0.088	0.061	0.001
	0	-0.3	20	100	0.045	0.054	0.029	0.036	0.035	0.041	0.073	0.083	0.076	0.193
			20	200	0.038	0.048	0.041	0.048	0.030	0.037	0.057	0.065	0.065	0.213
			40	100	0.026	0.033	0.014	0.017	0.016	0.020	0.052	0.072	0.073	0.257
			40	200	0.037	0.043	0.028	0.030	0.024	0.028	0.054	0.054	0.058	0.293
Smooth	0	0	20	100	0.054	0.064	0.042	0.050	0.040	0.047	0.090	0.115	0.014	0.010
			20	200	0.048	0.057	0.042	0.049	0.040	0.050	0.064	0.078	0.000	0.000
			40	100	0.053	0.062	0.032	0.038	0.037	0.042	0.096	0.125	0.103	0.078
			40	200	0.045	0.056	0.028	0.033	0.040	0.047	0.070	0.068	0.000	0.000

	0.3	0	20	100	0.039	0.050	0.041	0.052	0.041	0.049	0.058	0.111	0.027	0.020
			20	200	0.048	0.059	0.044	0.052	0.052	0.063	0.067	0.082	0.000	0.000
			40	100	0.032	0.034	0.032	0.039	0.025	0.031	0.045	0.117	0.122	0.100
			40	200	0.039	0.046	0.042	0.047	0.046	0.052	0.058	0.089	0.000	0.000
	-0.3	0	20	100	0.031	0.040	0.024	0.029	0.024	0.031	0.042	0.061	0.000	0.001
			20	200	0.042	0.049	0.042	0.048	0.033	0.040	0.050	0.065	0.000	0.000
			40	100	0.027	0.030	0.017	0.021	0.019	0.023	0.032	0.058	0.001	0.003
			40	200	0.040	0.045	0.037	0.044	0.026	0.029	0.051	0.066	0.000	0.000
	0	0.3	20	100	0.052	0.065	0.050	0.058	0.057	0.069	0.093	0.133	0.040	0.005
			20	200	0.052	0.064	0.048	0.056	0.055	0.069	0.071	0.087	0.000	0.000
			40	100	0.051	0.057	0.042	0.047	0.059	0.067	0.101	0.156	0.186	0.036
			40	200	0.046	0.055	0.039	0.049	0.060	0.067	0.075	0.089	0.000	0.000
	0	-0.3	20	100	0.035	0.041	0.023	0.029	0.025	0.032	0.057	0.062	0.001	0.001
			20	200	0.046	0.059	0.041	0.048	0.040	0.048	0.069	0.068	0.000	0.000
			40	100	0.030	0.034	0.014	0.019	0.018	0.021	0.053	0.058	0.002	0.003
			40	200	0.038	0.042	0.024	0.030	0.026	0.029	0.052	0.048	0.000	0.000
Mixed	0	0	20	100	0.057	0.066	0.046	0.056	0.050	0.057	0.095	0.127	0.000	0.000
			20	200	0.045	0.053	0.040	0.048	0.045	0.054	0.064	0.076	0.002	0.002
			40	100	0.050	0.058	0.033	0.038	0.033	0.036	0.088	0.128	0.000	0.000
			40	200	0.043	0.050	0.037	0.044	0.043	0.050	0.072	0.075	0.000	0.000
	0.3	0	20	100	0.042	0.050	0.044	0.052	0.041	0.051	0.063	0.105	0.001	0.001
			20	200	0.046	0.053	0.045	0.053	0.050	0.061	0.061	0.075	0.004	0.004
			40	100	0.027	0.032	0.036	0.041	0.026	0.029	0.051	0.125	0.000	0.000
			40	200	0.036	0.041	0.036	0.041	0.043	0.049	0.050	0.074	0.001	0.001
	-0.3	0	20	100	0.030	0.038	0.026	0.034	0.026	0.033	0.050	0.075	0.000	0.001
			20	200	0.038	0.048	0.034	0.041	0.028	0.034	0.050	0.062	0.000	0.002
			40	100	0.022	0.024	0.019	0.023	0.014	0.016	0.034	0.069	0.000	0.000
			40	200	0.038	0.045	0.032	0.040	0.027	0.032	0.056	0.069	0.000	0.000
	0	0.3	20	100	0.055	0.064	0.044	0.050	0.054	0.066	0.090	0.130	0.004	0.000
			20	200	0.042	0.049	0.040	0.050	0.053	0.064	0.071	0.066	0.006	0.000
			40	100	0.041	0.046	0.034	0.041	0.044	0.050	0.086	0.158	0.000	0.000
			40	200	0.044	0.051	0.036	0.041	0.052	0.059	0.063	0.072	0.001	0.000
	0	-0.3	20	100	0.031	0.037	0.019	0.024	0.022	0.026	0.052	0.052	0.000	0.000
			20	200	0.039	0.048	0.032	0.039	0.028	0.036	0.058	0.058	0.000	0.003
			40	100	0.024	0.026	0.009	0.013	0.017	0.020	0.045	0.054	0.000	0.000
			40	200	0.033	0.040	0.020	0.023	0.027	0.030	0.052	0.043	0.000	0.000

Notes: See Table 3.2.

Table 7.26. Power properties of panel statistics with $r_{max} = 4$

B. Type	θ	φ	N	T	$P(1)$	$P_M(1)$	$P(2)$	$P_M(2)$	$P(\hat{m}_i)$	$P_M(\hat{m}_i)$	$Z_\tau(1)$	$Z_\tau(2)$	Z_τ	Z_ϕ
No break	0	0	20	100	0.571	0.606	0.229	0.257	0.463	0.495	0.785	0.479	0.965	0.962
			20	200	1.000	1.000	0.957	0.963	0.966	0.969	1.000	0.994	1.000	1.000
			40	100	0.822	0.843	0.315	0.344	0.679	0.698	0.957	0.702	1.000	0.999
			40	200	1.000	1.000	0.999	0.999	0.991	0.991	1.000	1.000	1.000	1.000
	0.3	0	20	100	0.261	0.293	0.115	0.128	0.220	0.251	0.406	0.263	0.904	0.911
			20	200	0.994	0.996	0.761	0.790	0.945	0.953	0.999	0.909	1.000	1.000
			40	100	0.418	0.445	0.127	0.143	0.304	0.325	0.631	0.374	0.997	0.997
			40	200	1.000	1.000	0.955	0.962	0.984	0.986	1.000	0.996	1.000	1.000
	-0.3	0	20	100	0.251	0.282	0.092	0.108	0.194	0.217	0.384	0.224	0.634	0.823
			20	200	0.994	0.996	0.791	0.819	0.880	0.885	0.999	0.932	0.998	1.000
			40	100	0.405	0.435	0.112	0.125	0.312	0.337	0.596	0.323	0.828	0.968
			40	200	1.000	1.000	0.968	0.973	0.918	0.923	1.000	0.997	1.000	1.000
	0	0.3	20	100	0.526	0.559	0.218	0.242	0.435	0.471	0.743	0.466	0.980	0.786
			20	200	1.000	1.000	0.937	0.950	0.985	0.987	1.000	0.993	1.000	1.000
			40	100	0.787	0.807	0.322	0.339	0.647	0.669	0.943	0.701	1.000	0.977
			40	200	1.000	1.000	0.999	0.999	0.997	0.997	1.000	1.000	1.000	1.000
	0	-0.3	20	100	0.498	0.538	0.163	0.185	0.404	0.439	0.720	0.384	0.811	0.938
			20	200	1.000	1.000	0.954	0.963	0.947	0.952	1.000	0.993	1.000	1.000
			40	100	0.753	0.780	0.231	0.252	0.595	0.619	0.927	0.588	0.980	0.999
			40	200	1.000	1.000	0.998	0.998	0.979	0.979	1.000	1.000	1.000	1.000
Sharp	0	0	20	100	0.541	0.578	0.223	0.249	0.411	0.442	0.758	0.463	0.982	0.971
			20	200	0.999	0.999	0.931	0.941	0.956	0.962	1.000	0.989	1.000	1.000
			40	100	0.799	0.819	0.318	0.340	0.597	0.614	0.952	0.690	1.000	1.000
			40	200	1.000	1.000	0.999	0.999	0.985	0.987	1.000	1.000	1.000	1.000
	0.3	0	20	100	0.265	0.293	0.105	0.120	0.198	0.224	0.406	0.265	0.943	0.941
			20	200	0.984	0.988	0.707	0.736	0.905	0.916	0.996	0.875	1.000	1.000
			40	100	0.429	0.459	0.140	0.155	0.289	0.314	0.638	0.385	0.997	0.995
			40	200	1.000	1.000	0.949	0.957	0.978	0.979	1.000	0.995	1.000	1.000
	-0.3	0	20	100	0.249	0.277	0.097	0.110	0.184	0.210	0.370	0.214	0.742	0.883
			20	200	0.934	0.944	0.629	0.664	0.802	0.816	0.981	0.813	0.999	1.000
			40	100	0.370	0.400	0.099	0.112	0.255	0.276	0.561	0.304	0.970	0.998
			40	200	1.000	1.000	0.954	0.961	0.913	0.915	1.000	0.994	1.000	1.000
	0	0.3	20	100	0.488	0.521	0.190	0.214	0.380	0.420	0.705	0.443	0.982	0.810
			20	200	0.999	1.000	0.899	0.912	0.981	0.985	1.000	0.978	1.000	1.000
			40	100	0.759	0.784	0.284	0.310	0.581	0.606	0.935	0.663	1.000	0.987
			40	200	1.000	1.000	0.999	0.999	0.998	0.998	1.000	1.000	1.000	1.000
	0	-0.3	20	100	0.442	0.483	0.162	0.186	0.326	0.350	0.665	0.356	0.918	0.986
			20	200	0.995	0.996	0.838	0.857	0.913	0.919	0.999	0.954	1.000	1.000
			40	100	0.725	0.750	0.204	0.231	0.528	0.547	0.915	0.551	0.995	1.000
			40	200	1.000	1.000	0.999	0.999	0.959	0.962	1.000	1.000	1.000	1.000
Smooth	0	0	20	100	0.565	0.603	0.204	0.228	0.444	0.477	0.774	0.460	0.002	0.002
			20	200	1.000	1.000	0.958	0.967	0.974	0.976	1.000	0.994	0.001	0.007
			40	100	0.807	0.827	0.292	0.315	0.641	0.661	0.958	0.664	0.002	0.002
			40	200	1.000	1.000	0.998	0.999	0.991	0.992	1.000	1.000	0.000	0.000

	0.3	0	20	100	0.257	0.294	0.105	0.126	0.200	0.229	0.413	0.252	0.002	0.002
			20	200	0.994	0.997	0.764	0.792	0.934	0.941	0.999	0.912	0.009	0.019
			40	100	0.432	0.460	0.124	0.143	0.309	0.335	0.649	0.372	0.003	0.003
			40	200	1.000	1.000	0.955	0.961	0.984	0.985	1.000	0.994	0.000	0.002
	-0.3	0	20	100	0.259	0.291	0.075	0.091	0.201	0.226	0.394	0.187	0.000	0.000
			20	200	0.993	0.993	0.781	0.807	0.870	0.876	0.999	0.920	0.000	0.001
			40	100	0.408	0.434	0.093	0.104	0.300	0.324	0.593	0.276	0.000	0.000
			40	200	1.000	1.000	0.966	0.970	0.927	0.930	1.000	0.995	0.000	0.000
	0	0.3	20	100	0.503	0.538	0.207	0.231	0.412	0.452	0.724	0.451	0.007	0.006
			20	200	1.000	1.000	0.941	0.949	0.985	0.987	1.000	0.990	0.047	0.015
			40	100	0.771	0.794	0.295	0.319	0.637	0.663	0.933	0.674	0.007	0.006
			40	200	1.000	1.000	0.998	0.998	0.998	0.998	1.000	1.000	0.009	0.001
	0	-0.3	20	100	0.492	0.529	0.146	0.167	0.385	0.421	0.712	0.359	0.000	0.000
			20	200	1.000	1.000	0.950	0.958	0.950	0.952	1.000	0.995	0.000	0.003
			40	100	0.767	0.791	0.210	0.229	0.587	0.609	0.939	0.540	0.000	0.000
			40	200	1.000	1.000	1.000	1.000	0.973	0.975	1.000	1.000	0.000	0.000
Mixed	0	0	20	100	0.496	0.530	0.187	0.206	0.399	0.430	0.702	0.424	0.006	0.049
			20	200	1.000	1.000	0.930	0.941	0.960	0.964	1.000	0.987	0.872	0.948
			40	100	0.747	0.765	0.280	0.307	0.596	0.617	0.931	0.659	0.003	0.062
			40	200	1.000	1.000	0.998	0.999	0.989	0.991	1.000	1.000	0.998	1.000
	0.3	0	20	100	0.244	0.275	0.105	0.123	0.203	0.225	0.388	0.249	0.029	0.050
			20	200	0.988	0.991	0.720	0.752	0.919	0.932	0.998	0.892	0.874	0.952
			40	100	0.382	0.411	0.124	0.143	0.278	0.298	0.589	0.367	0.039	0.082
			40	200	1.000	1.000	0.939	0.946	0.980	0.983	1.000	0.992	0.998	1.000
	-0.3	0	20	100	0.185	0.206	0.070	0.081	0.137	0.156	0.293	0.170	0.000	0.013
			20	200	0.969	0.977	0.676	0.712	0.839	0.854	0.994	0.861	0.060	0.488
			40	100	0.275	0.296	0.070	0.084	0.198	0.219	0.464	0.257	0.000	0.012
			40	200	1.000	1.000	0.923	0.934	0.901	0.906	1.000	0.989	0.136	0.860
	0	0.3	20	100	0.457	0.491	0.181	0.201	0.391	0.425	0.669	0.407	0.094	0.025
			20	200	0.999	0.999	0.915	0.927	0.979	0.982	1.000	0.983	0.998	0.942
			40	100	0.704	0.728	0.256	0.285	0.565	0.593	0.911	0.622	0.144	0.019
			40	200	1.000	1.000	0.996	0.997	0.998	0.998	1.000	1.000	1.000	0.999
	0	-0.3	20	100	0.359	0.401	0.121	0.138	0.280	0.315	0.591	0.308	0.001	0.039
			20	200	0.998	0.999	0.886	0.904	0.933	0.937	0.999	0.975	0.273	0.841
			40	100	0.589	0.619	0.151	0.168	0.452	0.481	0.842	0.460	0.000	0.049
			40	200	1.000	1.000	0.993	0.995	0.962	0.964	1.000	1.000	0.651	0.995

Notes: See Table 3.2.

Table 7.27. Size properties of panel statistics with $r_{max} = 6$

B. Type	θ	φ	N	T	$P(1)$	$P_M(1)$	$P(2)$	$P_M(2)$	$P(\hat{m}_i)$	$P_M(\hat{m}_i)$	$Z_\tau(1)$	$Z_\tau(2)$	Z_τ	Z_ϕ
No break	0	0	20	100	0.060	0.067	0.048	0.057	0.057	0.064	0.095	0.122	0.110	0.072
			20	200	0.053	0.065	0.044	0.052	0.045	0.057	0.084	0.079	0.086	0.067
			40	100	0.053	0.062	0.030	0.037	0.045	0.050	0.093	0.134	0.123	0.071
			40	200	0.039	0.042	0.030	0.036	0.042	0.047	0.068	0.066	0.083	0.058
	0.3	0	20	100	0.046	0.055	0.050	0.062	0.045	0.052	0.066	0.116	0.097	0.069
			20	200	0.041	0.053	0.044	0.051	0.050	0.060	0.057	0.078	0.076	0.045
			40	100	0.027	0.035	0.034	0.043	0.026	0.030	0.049	0.131	0.101	0.054
			40	200	0.036	0.040	0.038	0.045	0.039	0.046	0.054	0.086	0.072	0.033
	-0.3	0	20	100	0.027	0.036	0.028	0.032	0.019	0.026	0.047	0.073	0.098	0.193
			20	200	0.046	0.054	0.041	0.051	0.035	0.042	0.060	0.067	0.081	0.198
			40	100	0.024	0.028	0.022	0.024	0.017	0.021	0.033	0.078	0.087	0.242
			40	200	0.036	0.043	0.037	0.043	0.027	0.032	0.049	0.065	0.070	0.262
	0	0.3	20	100	0.052	0.063	0.050	0.061	0.059	0.070	0.091	0.144	0.078	0.004
			20	200	0.045	0.055	0.043	0.052	0.061	0.069	0.068	0.084	0.069	0.002
			40	100	0.044	0.049	0.039	0.043	0.049	0.057	0.088	0.165	0.069	0.001
			40	200	0.046	0.051	0.043	0.047	0.061	0.071	0.073	0.091	0.052	0.000
	0	-0.3	20	100	0.039	0.047	0.031	0.035	0.029	0.035	0.069	0.077	0.073	0.175
			20	200	0.047	0.056	0.040	0.048	0.034	0.041	0.065	0.065	0.058	0.214
			40	100	0.033	0.042	0.019	0.023	0.022	0.027	0.070	0.068	0.074	0.241
			40	200	0.038	0.043	0.025	0.030	0.027	0.033	0.054	0.055	0.058	0.280
Sharp	0	0	20	100	0.054	0.069	0.048	0.055	0.045	0.054	0.097	0.128	0.106	0.071
			20	200	0.059	0.067	0.044	0.054	0.050	0.059	0.080	0.082	0.092	0.072
			40	100	0.044	0.048	0.034	0.041	0.037	0.044	0.087	0.126	0.117	0.065
			40	200	0.046	0.051	0.037	0.046	0.036	0.042	0.071	0.086	0.085	0.062
	0.3	0	20	100	0.039	0.048	0.042	0.050	0.044	0.050	0.061	0.112	0.086	0.063
			20	200	0.046	0.055	0.039	0.050	0.049	0.062	0.060	0.080	0.075	0.043
			40	100	0.034	0.039	0.039	0.048	0.027	0.031	0.054	0.137	0.101	0.056
			40	200	0.037	0.041	0.040	0.046	0.042	0.047	0.053	0.094	0.080	0.038
	-0.3	0	20	100	0.034	0.041	0.032	0.038	0.027	0.031	0.051	0.086	0.081	0.190
			20	200	0.049	0.056	0.047	0.053	0.032	0.037	0.062	0.073	0.073	0.198
			40	100	0.028	0.033	0.022	0.025	0.017	0.019	0.039	0.078	0.089	0.277
			40	200	0.037	0.043	0.034	0.040	0.026	0.029	0.049	0.063	0.068	0.261
	0	0.3	20	100	0.050	0.062	0.044	0.053	0.051	0.062	0.090	0.136	0.081	0.003
			20	200	0.044	0.056	0.040	0.049	0.048	0.059	0.074	0.079	0.067	0.004
			40	100	0.043	0.047	0.042	0.047	0.041	0.047	0.091	0.159	0.090	0.000
			40	200	0.039	0.048	0.043	0.051	0.056	0.063	0.065	0.093	0.061	0.001
	0	-0.3	20	100	0.036	0.044	0.029	0.037	0.029	0.036	0.059	0.078	0.079	0.194
			20	200	0.042	0.053	0.033	0.037	0.033	0.041	0.061	0.058	0.064	0.214
			40	100	0.032	0.036	0.018	0.021	0.020	0.023	0.056	0.072	0.068	0.253
			40	200	0.036	0.043	0.027	0.032	0.026	0.031	0.054	0.056	0.059	0.305
Smooth	0	0	20	100	0.055	0.065	0.038	0.048	0.047	0.056	0.098	0.105	0.014	0.011
			20	200	0.049	0.058	0.043	0.052	0.043	0.054	0.073	0.083	0.000	0.000
			40	100	0.051	0.058	0.027	0.032	0.031	0.036	0.089	0.115	0.099	0.073
			40	200	0.052	0.059	0.036	0.041	0.040	0.043	0.075	0.072	0.000	0.000

	0.3	0	20	100	0.040	0.048	0.040	0.047	0.038	0.048	0.060	0.102	0.022	0.015
			20	200	0.042	0.051	0.045	0.053	0.048	0.058	0.063	0.075	0.000	0.000
			40	100	0.032	0.040	0.031	0.037	0.030	0.032	0.051	0.117	0.116	0.098
			40	200	0.033	0.041	0.032	0.039	0.048	0.053	0.049	0.078	0.000	0.000
	-0.3	0	20	100	0.025	0.033	0.025	0.029	0.021	0.025	0.038	0.063	0.000	0.002
			20	200	0.042	0.051	0.043	0.050	0.034	0.040	0.053	0.070	0.000	0.000
			40	100	0.023	0.028	0.016	0.021	0.014	0.017	0.031	0.060	0.001	0.002
			40	200	0.042	0.047	0.036	0.041	0.031	0.035	0.052	0.067	0.000	0.000
	0	0.3	20	100	0.054	0.065	0.048	0.058	0.070	0.080	0.092	0.126	0.041	0.005
			20	200	0.051	0.061	0.050	0.060	0.070	0.079	0.085	0.089	0.000	0.000
			40	100	0.046	0.053	0.039	0.044	0.057	0.064	0.096	0.173	0.180	0.040
			40	200	0.049	0.056	0.046	0.054	0.064	0.077	0.075	0.097	0.000	0.000
	0	-0.3	20	100	0.043	0.050	0.029	0.035	0.026	0.032	0.060	0.067	0.000	0.000
			20	200	0.051	0.060	0.035	0.042	0.039	0.044	0.067	0.063	0.000	0.000
			40	100	0.026	0.030	0.016	0.017	0.017	0.022	0.055	0.058	0.003	0.006
			40	200	0.035	0.042	0.028	0.032	0.021	0.027	0.055	0.057	0.000	0.000
Mixed	0	0	20	100	0.049	0.057	0.037	0.043	0.046	0.056	0.082	0.103	0.000	0.001
			20	200	0.056	0.063	0.048	0.058	0.051	0.060	0.072	0.080	0.001	0.002
			40	100	0.047	0.051	0.034	0.039	0.036	0.043	0.086	0.130	0.000	0.000
			40	200	0.045	0.052	0.038	0.045	0.034	0.039	0.066	0.076	0.000	0.001
	0.3	0	20	100	0.039	0.045	0.034	0.043	0.039	0.046	0.054	0.104	0.001	0.001
			20	200	0.042	0.050	0.050	0.060	0.054	0.065	0.061	0.086	0.004	0.004
			40	100	0.027	0.030	0.030	0.036	0.026	0.032	0.045	0.120	0.000	0.000
			40	200	0.042	0.049	0.036	0.042	0.041	0.047	0.056	0.084	0.001	0.001
	-0.3	0	20	100	0.023	0.030	0.022	0.026	0.021	0.027	0.042	0.069	0.000	0.000
			20	200	0.042	0.049	0.035	0.044	0.034	0.041	0.056	0.060	0.000	0.002
			40	100	0.017	0.020	0.019	0.023	0.015	0.017	0.028	0.070	0.000	0.000
			40	200	0.027	0.033	0.024	0.027	0.022	0.027	0.043	0.057	0.000	0.000
	0	0.3	20	100	0.044	0.055	0.037	0.045	0.050	0.060	0.077	0.112	0.001	0.000
			20	200	0.038	0.047	0.037	0.042	0.061	0.073	0.061	0.073	0.004	0.000
			40	100	0.037	0.042	0.031	0.036	0.041	0.046	0.091	0.135	0.000	0.000
			40	200	0.038	0.041	0.026	0.030	0.045	0.053	0.056	0.069	0.001	0.000
	0	-0.3	20	100	0.032	0.042	0.019	0.024	0.026	0.033	0.051	0.062	0.000	0.001
			20	200	0.035	0.042	0.030	0.033	0.028	0.035	0.058	0.048	0.000	0.003
			40	100	0.027	0.031	0.014	0.018	0.021	0.026	0.045	0.066	0.000	0.000
			40	200	0.028	0.033	0.019	0.022	0.022	0.027	0.050	0.045	0.000	0.000

Notes: See Table 3.2.

Table 7.28. Power properties of panel statistics with $r_{max} = 6$

B. Type	θ	φ	N	T	$P(1)$	$P_M(1)$	$P(2)$	$P_M(2)$	$P(\hat{m}_i)$	$P_M(\hat{m}_i)$	$Z_\tau(1)$	$Z_\tau(2)$	Z_τ	Z_ϕ
No break	0	0	20	100	0.575	0.611	0.219	0.242	0.454	0.487	0.775	0.470	0.961	0.942
			20	200	1.000	1.000	0.955	0.963	0.966	0.970	1.000	0.992	1.000	1.000
			40	100	0.827	0.844	0.330	0.354	0.677	0.697	0.963	0.723	0.999	0.999
			40	200	1.000	1.000	1.000	1.000	0.992	0.992	1.000	1.000	1.000	1.000
	0.3	0	20	100	0.273	0.301	0.111	0.135	0.223	0.248	0.424	0.266	0.929	0.913
			20	200	0.994	0.996	0.749	0.780	0.935	0.943	0.999	0.908	1.000	1.000
			40	100	0.427	0.459	0.138	0.154	0.325	0.346	0.639	0.384	0.996	0.997
			40	200	1.000	1.000	0.959	0.965	0.981	0.982	1.000	0.997	1.000	1.000
	-0.3	0	20	100	0.274	0.300	0.086	0.103	0.213	0.237	0.400	0.217	0.621	0.830
			20	200	0.992	0.994	0.797	0.825	0.892	0.897	0.999	0.925	0.999	1.000
			40	100	0.392	0.419	0.108	0.120	0.295	0.318	0.587	0.321	0.846	0.979
			40	200	1.000	1.000	0.963	0.969	0.920	0.923	1.000	0.996	1.000	1.000
	0	0.3	20	100	0.510	0.546	0.207	0.233	0.435	0.471	0.727	0.457	0.981	0.817
			20	200	1.000	1.000	0.930	0.941	0.984	0.986	1.000	0.988	1.000	1.000
			40	100	0.772	0.793	0.300	0.325	0.661	0.683	0.943	0.709	1.000	0.968
			40	200	1.000	1.000	0.999	1.000	0.999	0.999	1.000	1.000	1.000	1.000
	0	-0.3	20	100	0.488	0.524	0.158	0.183	0.397	0.430	0.712	0.387	0.838	0.951
			20	200	1.000	1.000	0.949	0.957	0.951	0.954	1.000	0.992	1.000	1.000
			40	100	0.758	0.781	0.244	0.268	0.609	0.633	0.937	0.599	0.974	0.999
			40	200	1.000	1.000	0.999	0.999	0.977	0.977	1.000	1.000	1.000	1.000
Sharp	0	0	20	100	0.549	0.587	0.213	0.237	0.415	0.451	0.746	0.458	0.978	0.969
			20	200	0.999	0.999	0.929	0.941	0.953	0.957	1.000	0.985	1.000	1.000
			40	100	0.797	0.814	0.309	0.330	0.610	0.629	0.953	0.680	1.000	1.000
			40	200	1.000	1.000	0.999	0.999	0.988	0.989	1.000	1.000	1.000	1.000
	0.3	0	20	100	0.270	0.307	0.121	0.139	0.202	0.227	0.416	0.275	0.935	0.928
			20	200	0.985	0.990	0.714	0.743	0.909	0.922	0.999	0.873	1.000	1.000
			40	100	0.413	0.443	0.136	0.152	0.289	0.312	0.622	0.401	0.998	0.998
			40	200	1.000	1.000	0.947	0.954	0.975	0.977	1.000	0.996	1.000	1.000
	-0.3	0	20	100	0.239	0.267	0.094	0.114	0.168	0.193	0.365	0.223	0.726	0.898
			20	200	0.920	0.933	0.613	0.648	0.773	0.793	0.971	0.798	0.999	1.000
			40	100	0.381	0.408	0.106	0.116	0.248	0.270	0.581	0.302	0.952	0.995
			40	200	1.000	1.000	0.963	0.970	0.907	0.910	1.000	0.993	1.000	1.000
	0	0.3	20	100	0.484	0.517	0.197	0.226	0.376	0.410	0.702	0.439	0.986	0.814
			20	200	0.999	1.000	0.899	0.914	0.979	0.981	1.000	0.981	1.000	1.000
			40	100	0.764	0.787	0.281	0.306	0.595	0.622	0.938	0.671	1.000	0.991
			40	200	1.000	1.000	0.997	0.998	0.997	0.998	1.000	1.000	1.000	1.000
	0	-0.3	20	100	0.457	0.483	0.157	0.176	0.333	0.360	0.676	0.358	0.928	0.983
			20	200	0.989	0.993	0.836	0.855	0.904	0.913	0.998	0.947	1.000	1.000
			40	100	0.721	0.743	0.217	0.237	0.525	0.549	0.914	0.566	0.997	1.000
			40	200	1.000	1.000	0.998	0.999	0.961	0.963	1.000	1.000	1.000	1.000
Smooth	0	0	20	100	0.580	0.619	0.203	0.232	0.457	0.486	0.778	0.465	0.002	0.002
			20	200	1.000	1.000	0.962	0.970	0.969	0.971	1.000	0.995	0.002	0.004
			40	100	0.827	0.845	0.297	0.321	0.641	0.664	0.964	0.677	0.003	0.003
			40	200	1.000	1.000	1.000	1.000	0.990	0.991	1.000	1.000	0.000	0.000

	0.3	0	20	100	0.256	0.284	0.095	0.115	0.204	0.232	0.401	0.240	0.002	0.002
			20	200	0.994	0.996	0.759	0.787	0.933	0.943	0.999	0.906	0.007	0.020
			40	100	0.418	0.446	0.122	0.135	0.312	0.337	0.620	0.353	0.002	0.002
			40	200	1.000	1.000	0.955	0.962	0.981	0.983	1.000	0.997	0.000	0.001
	-0.3	0	20	100	0.243	0.277	0.085	0.101	0.190	0.214	0.371	0.193	0.000	0.000
			20	200	0.994	0.996	0.774	0.804	0.888	0.893	1.000	0.919	0.000	0.000
			40	100	0.400	0.426	0.088	0.101	0.299	0.320	0.591	0.268	0.000	0.000
			40	200	1.000	1.000	0.965	0.970	0.926	0.930	1.000	0.997	0.000	0.000
	0	0.3	20	100	0.510	0.546	0.201	0.224	0.428	0.463	0.727	0.454	0.005	0.004
			20	200	1.000	1.000	0.935	0.944	0.987	0.990	1.000	0.990	0.050	0.020
			40	100	0.798	0.818	0.310	0.336	0.646	0.674	0.954	0.700	0.006	0.005
			40	200	1.000	1.000	0.998	0.998	0.999	1.000	1.000	1.000	0.008	0.001
	0	-0.3	20	100	0.507	0.545	0.149	0.168	0.389	0.421	0.710	0.352	0.000	0.000
			20	200	1.000	1.000	0.944	0.957	0.941	0.944	1.000	0.993	0.000	0.004
			40	100	0.749	0.768	0.212	0.230	0.578	0.596	0.928	0.538	0.000	0.000
			40	200	1.000	1.000	0.999	1.000	0.971	0.973	1.000	1.000	0.000	0.000
Mixed	0	0	20	100	0.490	0.525	0.193	0.212	0.408	0.441	0.698	0.432	0.011	0.051
			20	200	1.000	1.000	0.929	0.943	0.966	0.969	1.000	0.987	0.856	0.936
			40	100	0.760	0.779	0.288	0.316	0.595	0.619	0.928	0.670	0.003	0.046
			40	200	1.000	1.000	0.999	1.000	0.986	0.988	1.000	1.000	0.996	0.999
	0.3	0	20	100	0.237	0.266	0.100	0.119	0.198	0.221	0.375	0.239	0.039	0.078
			20	200	0.987	0.991	0.727	0.757	0.921	0.930	0.997	0.887	0.900	0.953
			40	100	0.400	0.428	0.123	0.137	0.286	0.309	0.603	0.375	0.045	0.105
			40	200	1.000	1.000	0.945	0.953	0.977	0.979	1.000	0.996	0.998	1.000
	-0.3	0	20	100	0.196	0.219	0.083	0.098	0.156	0.173	0.303	0.198	0.000	0.011
			20	200	0.973	0.977	0.687	0.716	0.852	0.862	0.994	0.860	0.057	0.523
			40	100	0.280	0.302	0.080	0.087	0.214	0.231	0.449	0.263	0.000	0.016
			40	200	1.000	1.000	0.922	0.933	0.909	0.913	1.000	0.991	0.119	0.882
	0	0.3	20	100	0.452	0.492	0.168	0.191	0.387	0.422	0.684	0.407	0.098	0.017
			20	200	0.999	0.999	0.920	0.934	0.980	0.983	1.000	0.986	0.999	0.955
			40	100	0.701	0.727	0.260	0.281	0.577	0.602	0.911	0.631	0.122	0.015
			40	200	1.000	1.000	0.997	0.997	0.998	0.998	1.000	1.000	1.000	1.000
	0	-0.3	20	100	0.383	0.414	0.117	0.137	0.296	0.327	0.580	0.300	0.001	0.047
			20	200	1.000	1.000	0.895	0.914	0.935	0.939	1.000	0.980	0.258	0.856
			40	100	0.593	0.622	0.157	0.170	0.443	0.466	0.835	0.467	0.000	0.074
			40	200	1.000	1.000	0.993	0.995	0.961	0.965	1.000	1.000	0.600	0.995

Notes: See Table 3.2.

Table 7.29. Size properties of panel statistics with $r_{max} = 7$

B. Type	θ	φ	N	T	$P(1)$	$P_M(1)$	$P(2)$	$P_M(2)$	$P(\hat{m}_i)$	$P_M(\hat{m}_i)$	$Z_\tau(1)$	$Z_\tau(2)$	Z_τ	Z_ϕ
No break	0	0	20	100	0.052	0.063	0.043	0.051	0.042	0.050	0.086	0.112	0.122	0.079
			20	200	0.057	0.068	0.045	0.052	0.054	0.065	0.086	0.084	0.090	0.070
			40	100	0.043	0.050	0.036	0.043	0.033	0.037	0.089	0.144	0.119	0.073
			40	200	0.049	0.056	0.039	0.044	0.044	0.053	0.075	0.072	0.087	0.063
	0.3	0	20	100	0.036	0.044	0.044	0.053	0.036	0.045	0.055	0.106	0.108	0.075
			20	200	0.045	0.053	0.055	0.065	0.052	0.063	0.062	0.087	0.072	0.039
			40	100	0.031	0.036	0.036	0.040	0.029	0.034	0.050	0.122	0.104	0.058
			40	200	0.031	0.038	0.036	0.041	0.040	0.047	0.050	0.081	0.077	0.033
	-0.3	0	20	100	0.033	0.041	0.028	0.034	0.026	0.031	0.046	0.072	0.093	0.187
			20	200	0.044	0.053	0.043	0.051	0.033	0.041	0.058	0.073	0.069	0.193
			40	100	0.028	0.033	0.025	0.028	0.024	0.025	0.039	0.070	0.095	0.259
			40	200	0.038	0.045	0.030	0.037	0.023	0.028	0.050	0.062	0.075	0.254
	0	0.3	20	100	0.062	0.072	0.045	0.057	0.064	0.077	0.093	0.135	0.078	0.004
			20	200	0.053	0.061	0.046	0.054	0.072	0.082	0.079	0.087	0.065	0.005
			40	100	0.049	0.055	0.043	0.050	0.055	0.061	0.097	0.176	0.065	0.001
			40	200	0.045	0.054	0.041	0.047	0.059	0.067	0.074	0.086	0.058	0.000
	0	-0.3	20	100	0.044	0.051	0.031	0.039	0.035	0.042	0.067	0.081	0.070	0.184
			20	200	0.039	0.049	0.037	0.044	0.030	0.038	0.061	0.061	0.063	0.214
			40	100	0.034	0.039	0.018	0.021	0.023	0.028	0.060	0.074	0.065	0.240
			40	200	0.037	0.045	0.030	0.035	0.025	0.029	0.058	0.055	0.056	0.278
Sharp	0	0	20	100	0.063	0.075	0.043	0.053	0.048	0.056	0.099	0.123	0.106	0.068
			20	200	0.046	0.056	0.045	0.054	0.042	0.050	0.071	0.081	0.083	0.067
			40	100	0.053	0.062	0.032	0.038	0.038	0.041	0.093	0.137	0.110	0.068
			40	200	0.049	0.056	0.038	0.044	0.045	0.052	0.078	0.086	0.085	0.059
	0.3	0	20	100	0.045	0.054	0.041	0.050	0.038	0.045	0.070	0.115	0.096	0.063
			20	200	0.043	0.053	0.047	0.055	0.047	0.058	0.059	0.085	0.074	0.044
			40	100	0.029	0.036	0.036	0.041	0.028	0.035	0.051	0.131	0.097	0.046
			40	200	0.036	0.040	0.038	0.045	0.039	0.047	0.054	0.083	0.082	0.035
	-0.3	0	20	100	0.039	0.046	0.036	0.042	0.030	0.037	0.055	0.078	0.085	0.191
			20	200	0.042	0.049	0.042	0.051	0.032	0.039	0.054	0.068	0.077	0.194
			40	100	0.024	0.029	0.022	0.027	0.016	0.019	0.038	0.074	0.096	0.270
			40	200	0.044	0.051	0.034	0.037	0.030	0.035	0.055	0.069	0.073	0.264
	0	0.3	20	100	0.052	0.061	0.046	0.055	0.056	0.063	0.085	0.129	0.085	0.005
			20	200	0.040	0.050	0.043	0.051	0.059	0.069	0.062	0.072	0.070	0.003
			40	100	0.053	0.061	0.039	0.047	0.052	0.060	0.105	0.173	0.083	0.001
			40	200	0.040	0.047	0.042	0.049	0.054	0.066	0.073	0.097	0.063	0.000
	0	-0.3	20	100	0.041	0.047	0.026	0.033	0.029	0.034	0.068	0.085	0.071	0.189
			20	200	0.036	0.044	0.035	0.039	0.028	0.033	0.055	0.055	0.066	0.206
			40	100	0.033	0.038	0.019	0.021	0.021	0.025	0.050	0.067	0.070	0.256
			40	200	0.034	0.040	0.024	0.030	0.021	0.025	0.056	0.054	0.063	0.303
Smooth	0	0	20	100	0.048	0.058	0.038	0.049	0.044	0.052	0.087	0.109	0.013	0.008
			20	200	0.055	0.066	0.044	0.053	0.048	0.059	0.079	0.079	0.000	0.000
			40	100	0.047	0.055	0.028	0.033	0.037	0.044	0.096	0.116	0.099	0.072
			40	200	0.040	0.046	0.034	0.040	0.034	0.040	0.066	0.067	0.000	0.000

	0.3	0	20	100	0.038	0.043	0.036	0.047	0.040	0.050	0.055	0.107	0.024	0.016
			20	200	0.043	0.053	0.045	0.053	0.046	0.053	0.062	0.083	0.000	0.000
			40	100	0.032	0.037	0.037	0.043	0.032	0.039	0.044	0.123	0.120	0.102
			40	200	0.044	0.048	0.034	0.042	0.045	0.050	0.057	0.080	0.000	0.000
	-0.3	0	20	100	0.032	0.041	0.027	0.034	0.022	0.029	0.042	0.070	0.000	0.001
			20	200	0.040	0.054	0.045	0.053	0.030	0.039	0.054	0.077	0.000	0.000
			40	100	0.024	0.029	0.018	0.020	0.015	0.017	0.034	0.053	0.001	0.004
			40	200	0.031	0.039	0.033	0.038	0.022	0.026	0.047	0.065	0.000	0.000
	0	0.3	20	100	0.050	0.061	0.043	0.053	0.055	0.064	0.087	0.134	0.032	0.002
			20	200	0.049	0.060	0.048	0.060	0.072	0.083	0.076	0.092	0.000	0.000
			40	100	0.047	0.053	0.036	0.041	0.057	0.065	0.092	0.155	0.194	0.042
			40	200	0.048	0.054	0.043	0.048	0.061	0.071	0.080	0.092	0.000	0.000
	0	-0.3	20	100	0.038	0.047	0.025	0.032	0.031	0.038	0.060	0.063	0.001	0.001
			20	200	0.052	0.059	0.034	0.044	0.041	0.049	0.066	0.066	0.000	0.000
			40	100	0.034	0.040	0.017	0.021	0.023	0.026	0.054	0.061	0.003	0.005
			40	200	0.037	0.043	0.024	0.028	0.026	0.031	0.053	0.047	0.000	0.000
Mixed	0	0	20	100	0.050	0.060	0.037	0.044	0.046	0.056	0.090	0.110	0.000	0.000
			20	200	0.053	0.059	0.048	0.058	0.044	0.051	0.078	0.087	0.001	0.001
			40	100	0.047	0.053	0.031	0.036	0.038	0.044	0.089	0.132	0.000	0.000
			40	200	0.046	0.053	0.035	0.041	0.042	0.048	0.070	0.077	0.000	0.000
	0.3	0	20	100	0.036	0.046	0.043	0.049	0.037	0.047	0.061	0.108	0.002	0.002
			20	200	0.041	0.049	0.048	0.058	0.053	0.063	0.055	0.083	0.006	0.005
			40	100	0.029	0.035	0.032	0.041	0.028	0.032	0.051	0.120	0.000	0.000
			40	200	0.034	0.038	0.037	0.042	0.044	0.049	0.049	0.074	0.001	0.000
	-0.3	0	20	100	0.032	0.039	0.029	0.036	0.023	0.029	0.046	0.067	0.000	0.000
			20	200	0.040	0.046	0.035	0.041	0.031	0.036	0.055	0.054	0.000	0.002
			40	100	0.022	0.025	0.018	0.020	0.016	0.019	0.030	0.065	0.000	0.000
			40	200	0.032	0.040	0.031	0.035	0.025	0.030	0.047	0.062	0.000	0.000
	0	0.3	20	100	0.039	0.048	0.040	0.045	0.048	0.058	0.073	0.114	0.001	0.000
			20	200	0.039	0.048	0.036	0.044	0.055	0.066	0.064	0.068	0.006	0.000
			40	100	0.036	0.040	0.034	0.040	0.042	0.052	0.073	0.139	0.000	0.000
			40	200	0.041	0.046	0.035	0.040	0.055	0.065	0.069	0.077	0.001	0.000
	0	-0.3	20	100	0.033	0.041	0.021	0.027	0.027	0.032	0.053	0.059	0.000	0.000
			20	200	0.036	0.044	0.029	0.036	0.025	0.031	0.048	0.044	0.000	0.003
			40	100	0.019	0.024	0.010	0.013	0.015	0.017	0.042	0.053	0.000	0.000
			40	200	0.030	0.033	0.020	0.025	0.020	0.024	0.042	0.044	0.000	0.000

Notes: See Table 3.2.

Table 7.30. Power properties of panel statistics with $r_{max} = 7$

B. Type	θ	φ	N	T	$P(1)$	$P_M(1)$	$P(2)$	$P_M(2)$	$P(\hat{m}_i)$	$P_M(\hat{m}_i)$	$Z_\tau(1)$	$Z_\tau(2)$	Z_τ	Z_ϕ
No break	0	0	20	100	0.559	0.598	0.223	0.252	0.462	0.497	0.779	0.493	0.946	0.926
			20	200	1.000	1.000	0.954	0.963	0.978	0.981	1.000	0.996	1.000	1.000
			40	100	0.813	0.831	0.326	0.353	0.668	0.688	0.963	0.691	0.999	0.999
			40	200	1.000	1.000	1.000	1.000	0.992	0.992	1.000	1.000	1.000	1.000
	0.3	0	20	100	0.286	0.314	0.108	0.127	0.225	0.253	0.413	0.271	0.918	0.892
			20	200	0.994	0.995	0.765	0.792	0.936	0.945	1.000	0.912	1.000	1.000
			40	100	0.423	0.455	0.128	0.143	0.327	0.350	0.637	0.376	0.997	0.996
			40	200	1.000	1.000	0.956	0.962	0.984	0.985	1.000	0.997	1.000	1.000
	-0.3	0	20	100	0.255	0.283	0.092	0.112	0.194	0.221	0.380	0.217	0.572	0.804
			20	200	0.995	0.996	0.791	0.817	0.897	0.903	0.999	0.916	0.999	1.000
			40	100	0.398	0.424	0.103	0.118	0.299	0.321	0.587	0.313	0.867	0.980
			40	200	1.000	1.000	0.964	0.969	0.923	0.926	1.000	0.995	1.000	1.000
	0	0.3	20	100	0.524	0.565	0.217	0.246	0.449	0.482	0.738	0.483	0.975	0.765
			20	200	1.000	1.000	0.935	0.947	0.984	0.986	1.000	0.994	1.000	1.000
			40	100	0.784	0.805	0.303	0.328	0.664	0.688	0.946	0.701	0.999	0.969
			40	200	1.000	1.000	0.998	0.999	0.998	0.999	1.000	1.000	1.000	1.000
	0	-0.3	20	100	0.505	0.545	0.172	0.193	0.400	0.434	0.717	0.399	0.825	0.955
			20	200	1.000	1.000	0.951	0.960	0.944	0.950	1.000	0.991	1.000	1.000
			40	100	0.750	0.768	0.237	0.258	0.588	0.607	0.929	0.598	0.981	1.000
			40	200	1.000	1.000	1.000	1.000	0.969	0.972	1.000	1.000	1.000	1.000
Sharp	0	0	20	100	0.553	0.588	0.203	0.230	0.412	0.442	0.763	0.462	0.984	0.981
			20	200	0.999	1.000	0.936	0.946	0.961	0.965	1.000	0.989	1.000	1.000
			40	100	0.808	0.830	0.322	0.343	0.608	0.634	0.960	0.709	1.000	1.000
			40	200	1.000	1.000	0.999	0.999	0.986	0.986	1.000	1.000	1.000	1.000
	0.3	0	20	100	0.269	0.303	0.106	0.122	0.202	0.224	0.414	0.255	0.922	0.919
			20	200	0.986	0.988	0.722	0.758	0.919	0.929	0.997	0.890	1.000	1.000
			40	100	0.425	0.452	0.146	0.161	0.289	0.307	0.629	0.391	0.998	0.998
			40	200	1.000	1.000	0.955	0.960	0.977	0.980	1.000	0.996	1.000	1.000
	-0.3	0	20	100	0.251	0.281	0.094	0.112	0.188	0.215	0.374	0.224	0.730	0.905
			20	200	0.931	0.945	0.641	0.676	0.786	0.803	0.981	0.815	1.000	1.000
			40	100	0.381	0.407	0.116	0.132	0.261	0.282	0.565	0.331	0.970	0.998
			40	200	1.000	1.000	0.959	0.968	0.898	0.902	1.000	0.995	1.000	1.000
	0	0.3	20	100	0.485	0.530	0.195	0.218	0.387	0.418	0.709	0.446	0.979	0.816
			20	200	0.998	0.998	0.907	0.923	0.975	0.976	1.000	0.979	1.000	1.000
			40	100	0.759	0.783	0.281	0.307	0.587	0.612	0.935	0.685	1.000	0.985
			40	200	1.000	1.000	0.998	0.999	0.997	0.997	1.000	1.000	1.000	1.000
	0	-0.3	20	100	0.443	0.479	0.163	0.185	0.324	0.354	0.670	0.378	0.904	0.974
			20	200	0.994	0.996	0.843	0.862	0.909	0.916	0.999	0.954	1.000	1.000
			40	100	0.729	0.751	0.221	0.242	0.530	0.553	0.918	0.568	0.996	1.000
			40	200	1.000	1.000	0.998	0.999	0.961	0.964	1.000	1.000	1.000	1.000
Smooth	0	0	20	100	0.572	0.611	0.205	0.224	0.449	0.486	0.774	0.455	0.002	0.002
			20	200	1.000	1.000	0.960	0.968	0.972	0.974	1.000	0.994	0.003	0.006
			40	100	0.818	0.843	0.285	0.318	0.631	0.651	0.959	0.670	0.005	0.005
			40	200	1.000	1.000	0.999	1.000	0.990	0.991	1.000	1.000	0.000	0.000

	0.3	0	20	100	0.278	0.304	0.103	0.118	0.208	0.234	0.403	0.242	0.002	0.001
			20	200	0.994	0.996	0.748	0.780	0.934	0.944	0.999	0.910	0.007	0.013
			40	100	0.434	0.464	0.126	0.142	0.317	0.339	0.649	0.375	0.003	0.003
			40	200	1.000	1.000	0.960	0.967	0.982	0.983	1.000	0.993	0.000	0.000
	-0.3	0	20	100	0.270	0.301	0.087	0.100	0.211	0.232	0.399	0.205	0.000	0.000
			20	200	0.994	0.996	0.771	0.799	0.885	0.896	0.999	0.922	0.000	0.000
			40	100	0.408	0.434	0.101	0.111	0.290	0.315	0.597	0.268	0.000	0.000
			40	200	1.000	1.000	0.969	0.973	0.914	0.918	1.000	0.998	0.000	0.000
	0	0.3	20	100	0.520	0.559	0.193	0.216	0.432	0.464	0.734	0.446	0.004	0.003
			20	200	1.000	1.000	0.938	0.950	0.987	0.989	1.000	0.993	0.041	0.016
			40	100	0.790	0.807	0.296	0.320	0.645	0.668	0.947	0.696	0.004	0.004
			40	200	1.000	1.000	0.999	0.999	0.999	0.999	1.000	1.000	0.014	0.001
	0	-0.3	20	100	0.513	0.547	0.155	0.176	0.406	0.435	0.709	0.365	0.000	0.000
			20	200	1.000	1.000	0.944	0.953	0.938	0.941	1.000	0.992	0.000	0.003
			40	100	0.752	0.778	0.206	0.229	0.587	0.610	0.937	0.537	0.000	0.000
			40	200	1.000	1.000	0.999	0.999	0.969	0.971	1.000	1.000	0.000	0.000
Mixed	0	0	20	100	0.489	0.520	0.199	0.220	0.388	0.417	0.702	0.441	0.009	0.046
			20	200	1.000	1.000	0.932	0.944	0.965	0.969	1.000	0.990	0.889	0.949
			40	100	0.755	0.778	0.286	0.305	0.604	0.626	0.938	0.670	0.008	0.063
			40	200	1.000	1.000	0.998	0.998	0.989	0.990	1.000	1.000	0.995	0.999
	0.3	0	20	100	0.250	0.279	0.089	0.108	0.195	0.220	0.375	0.243	0.039	0.063
			20	200	0.989	0.990	0.727	0.761	0.923	0.934	0.998	0.898	0.884	0.952
			40	100	0.385	0.413	0.126	0.141	0.299	0.324	0.595	0.385	0.044	0.101
			40	200	1.000	1.000	0.945	0.952	0.980	0.982	1.000	0.995	0.996	1.000
	-0.3	0	20	100	0.173	0.202	0.072	0.089	0.130	0.153	0.287	0.185	0.001	0.020
			20	200	0.971	0.976	0.675	0.713	0.841	0.851	0.995	0.852	0.060	0.503
			40	100	0.291	0.309	0.077	0.086	0.213	0.232	0.450	0.255	0.000	0.013
			40	200	1.000	1.000	0.906	0.922	0.912	0.916	1.000	0.987	0.086	0.836
	0	0.3	20	100	0.463	0.495	0.186	0.208	0.372	0.407	0.674	0.415	0.089	0.012
			20	200	1.000	1.000	0.912	0.926	0.982	0.985	1.000	0.988	0.997	0.952
			40	100	0.709	0.732	0.248	0.272	0.574	0.600	0.911	0.629	0.172	0.024
			40	200	1.000	1.000	0.997	0.998	0.998	0.999	1.000	1.000	1.000	1.000
	0	-0.3	20	100	0.357	0.388	0.128	0.145	0.288	0.314	0.560	0.294	0.001	0.051
			20	200	0.999	0.999	0.903	0.916	0.928	0.933	1.000	0.981	0.285	0.839
			40	100	0.599	0.619	0.149	0.163	0.458	0.475	0.833	0.470	0.000	0.057
			40	200	1.000	1.000	0.995	0.996	0.959	0.962	1.000	1.000	0.638	0.994

Notes: See Table 3.2.