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Wave Scattering by a Planar Junction Between Anomalously Reflecting Metasurface and Impedance Sheets

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ABSTRACT Metasurface technology has become one of the promising structures for controlling and manipulating of the phase and amplitude characteristics of the electromagnetic waves in recent years. For this reason, the determination of the exact diffracted fields is also crucial especially in the edge or discontinuity points of these types of materials. In the paper, the discontinuity problem by the junction between the layers of an anomalously reflecting metasurface and an impedance half-screen is taken into consideration. These two planar layers have a common edge discontinuity where the diffracted waves occur. The scattered waves from the structure are obtained by the novel technique of the method of transition boundary, which especially offers the relation between the diffracted and scattered geometrical optical waves at the transition boundaries. The diffracted wave expressions are derived by the technique of the method of transition boundary and the uniform wave expressions are obtained by the method of uniform geometrical theory of diffraction. The total wave result of the method of transition boundary is also compared and verified by the well-known theory of physical optics. The behaviors of the total wave and its components are analyzed numerically.

INDEX TERMS Metasurfaces, anomalous reflection, diffraction theory, exact solution.

I. INTRODUCTION

The main difference in the anomalous reflection is the waves do not obey Fermat's principle. It is observed in the metamaterials and metasurfaces, which are artificial constructions and are generally constructed with a desired permittivity and permeability [1]. As a result of the increase in the number of studies in the field of the metamaterials and metasurfaces technology, these artificial constructions are also used to excite the anomalously reflected waves on the scattered waves [2]–[5]. The anomalous effects of these structures are used in a wide range for different purposes with different techniques such as forming metasurfaces by using several resonators, controlling of terahertz (THz) wave by using coding metasurfaces, controlling of both anomalous reflection and phase difference over supercells, creating an anomalous reflection by using cylindrical nanorods and forming gradient metasurfaces for anomalous reflection in optical frequencies [6]–[10]. These are some of the purposes for the use of metamaterials and metasurfaces.

The scattering problem is a phenomenon in the field of electromagnetics. Especially, the determination of the exact diffracted waves by the edge or discontinuity points of the scatterers [11]–[16] is very crucial for the control of the scattering waves or constructing the type of meta structures having electromagnetic invisibility with having reflectionless property or very low-scattering [17]–[19]. The scattering and diffraction problems on metasurfaces have also been studied by several authors in recent years [20]–[24]. Umul investigated the effect of the discontinuities on the anomalous reflection characteristics of metasurfaces in several studies and investigated the metasurface structures with the different discontinuity problems [24], [27]–[33]. After his several scattering problem studies, he has realized that the diffracted waves have a direct relation with the scattered geometrical optical (GO) waves at the transition boundaries. For this reason, Umul has transformed these studies into a novel technique, which is named as the method of transition boundary (MTB) [34]. The technique is applied

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to several scattering problems by several authors in several studies [35]–[43].

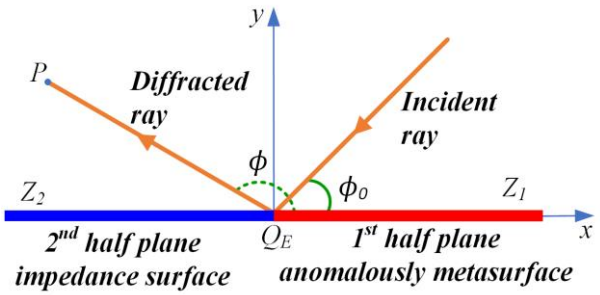

FIGURE 1. **The geometry of the scattering problem.**

In this paper, we will be investigating the scattering properties of waves by a discontinuous junction point between an anomalously reflecting metasurface screen and an impedance half-screen by the novel technique of the MTB [34]. To the best of our knowledge, such a kind of scenario has not been investigated yet. In the literature, the existing studies do not include such a junction discontinuity problem with the inclusion of an anomalously reflecting metasurface. First of all, the initial fields will be determined. Then, the total and scattered GO waves will be obtained for the problem under consideration. After that, the diffracted wave expression will be derived by the superposition of the diffracted fields. The uniform diffracted wave will be expressed by using the uniform geometrical theory of diffraction (UTD) [44]. The behaviors of the total, total GO, and diffraction field expressions will be analyzed numerically.

A time factor of $\exp(j\omega t)$ is assumed and suppressed throughout the paper. ω and t are the angular frequency and time, respectively.

II. DEFINITION OF THE PROBLEM

An anomalously reflecting metasurface half-plane and impedance half-plane which have a common planar edge discontinuity are located at $x \in (-\infty, 0]$ for the impedance half-plane and $x \in [0, \infty)$ for the anomalously reflecting metasurface half-plane as well as common locations for both at $y = 0$ and $z \in (-\infty, \infty)$. The surface impedances of half-screens are Z_1 and Z_2. The geometry of the problem is shown in Fig. 1. P is the observation point.

The incident plane wave of

$$u_i = u_0 e^{jk\rho\cos(\phi-\phi_0)} \tag{1}$$

is illuminating the screens. u_0 is the complex amplitude and and k is the wave number. u represents a component of the electromagnetic field. Q_E is the diffraction point of the screen. The cylindrical coordinates are given by (ρ, ϕ, z). ϕ_0 is the angle of incidence. The electric field of the incident wave is parallel to the z axis. For this reason, the incident wave in (1) is the electric field.

III. MTB SOLUTION OF THE PROBLEM

In order to determine the exact diffracted waves in our problem, we will use the novel technique of MTB that was introduced by Umul offering an easiness to determine exact diffracted waves [34]. The method is especially a competent technique for the determination of the diffracted waves by the discontinuities at the transition boundaries.

The diffracted wave has the general expression of

$$u_d = u_i(Q_E)\frac{e^{-j\frac{\pi}{4}}}{\sqrt{2\pi}}\frac{f(\alpha_0, \beta_E)}{\cos\beta_E + \cos\alpha_0}\frac{e^{-jkR_E}}{\sqrt{kR_E}} \tag{2}$$

at high-frequencies [45]. f is a function that will be determined. β_E and α_0 are the angles of observation and incidence, which are measured from the surface of the scatterer. R_E is the distance between the edge and observation points. The general geometry that defines (2) is given in Fig. 2.

The scattered GO field has the expression of

$$u_{SGO} = RU(\pi - \alpha_0 - \beta_E)u_r(P) + TU(\beta_E - \pi - \alpha_0)u_i(P) \tag{3}$$

for the geometry in Fig. 2. $U(x)$ is the unit-step function, which is equal to one for $x > 0$ and zero otherwise. R and T are the reflection and transmission coefficients, which are related with the boundary conditions on the scatterer. u_r is the reflected wave from the surface. There are two transition boundaries, which are located at $\pi - \alpha_0$ for the reflection boundary and $\pi + \alpha_0$ for the shadow boundary, respectively. The transition boundary relations for reflection and shadow boundaries can be written as

$$f(\pi - \alpha_0, \alpha_0) = -\sin\alpha_0 R \tag{4}$$

and

$$f(\pi + \alpha_0, \alpha_0) = -\sin\alpha_0 T, \tag{5}$$

respectively [46], [47].

We will apply the technique of MTB [34] consisting of five main steps to find the total wave and its components for our metasurface junction problem by taking into consideration these steps.

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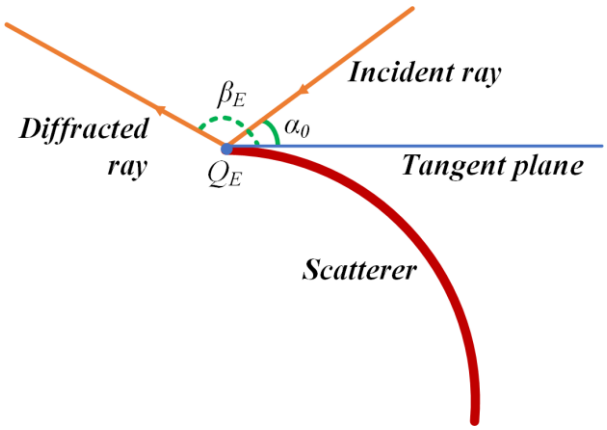

FIGURE 2. **Diffraction geometry of the scatterer.**

A. INITIAL WAVE

MTB offers the term of the initial wave (u_{in}) instead of the incident wave of u_i. The usage of this term has a great advantage to define the problem by excluding a part of the structure that causes discontinuity and diffraction. In the half-plane or wedge type construction problems, their presence directly causes discontinuity. For this reason, the initial waves are directly equal to the incident wave in these types of problems. However, in our problem, the discontinuity arises from the junction point of two different planes, and it is enough to exclude one of the half-screens for canceling the discontinuity. Therefore, the initial wave is different from the incident wave in our problem.

In order to determine the initial wave in our problem, we exclude the first half-plane of the anomalously reflecting metasurface with the impedance of Z_1 from the space. In this case, there will only be a whole impedance half-screen having the impedance of Z_2 at the plane of $y=0$.

The reflection coefficients of the first and second half-plane of R_1 and R_2 can be given as

$$R_1 = \frac{\sin\phi_0 - \sin\theta_1}{\sin\phi_r + \sin\theta_1} \tag{6}$$

and

$$R_2 = \frac{\sin\phi_0 - \sin\theta_2}{\sin\phi_0 + \sin\theta_2}, \tag{7}$$

respectively. $\sin\theta_i$ is equal to $Z_0 / 2R_{e(i)}$ where Z_0 is the impedance of the free space and $R_{e(i)}$ is the resistivity of the ith half-plane. $\sin\theta_i$ is a ratio that can be expressed as greater than one in some cases and the numerical results when the resistivity of the half-plane is especially greater than the impedance of the free space. ϕ_r is the angle of reflection that can be chosen as a different value from the incident angle of ϕ_0 for providing the anomalous reflection condition.

As a result, the initial wave can be defined by

$$u_{in}(P) = u_0 e^{jk\rho\cos(\phi-\phi_0)} + R_2 u_0 e^{jk\rho\cos(\phi+\phi_0)} \tag{8}$$

by excluding the first half-plane of the anomalously reflecting metasurface in this case by applying the first step of the MTB.

B. TOTAL GO WAVE

In order to determine the diffracted wave, the scattered GO wave must be first obtained. Before we obtain the scattered GO wave, the total GO wave must be obtained by considering the whole structure with the inclusion of both half-planes and applying the second step of the MTB. Hence, the general expression of the total GO wave can be written as

$$u_{TGO} = u_i U\left(-\xi_{0-}\right) + u_r U\left(-\xi_{1+}\right) \tag{9}$$

according to the geometry in Fig. 1. The detour parameters of $\xi_{0\pm}$ and $\xi_{1\pm}$ can be introduced by

$$\xi_{0\pm} = -\sqrt{2k\rho}\cos\frac{\phi \pm \phi_0}{2} \tag{10}$$

and

$$\xi_{1\pm} = -\sqrt{2k\rho}\cos\frac{\phi \pm \phi_r}{2}, \tag{11}$$

respectively. The total GO wave reads

$$u_{TGO}(P) = u_0 e^{jk\rho\cos(\phi-\phi_0)} + R_1 u_r U\left(-\xi_{1+}\right) + R_2 u_0 e^{jk\rho\cos(\phi+\phi_0)} U\left(\xi_{0+}\right) \tag{12}$$

where u_r can be written as

$$u_r = u_0 e^{jk\left[x_s(\cos\phi_0 - \cos\phi_r) + \rho\cos(\phi+\phi_r)\right]} \tag{13}$$

where x_s is the coordinate of the reflection point which can be defined by

$$x_s = \frac{\rho\sin\left(\phi+\phi_r\right)}{\sin\phi_r}. \tag{14}$$

According to (14), a phase shift accompanies the amplitude variation of the reflected wave [29].

C. SCATTERED GO WAVE

The scattered GO wave is found by

$$u_{SGO} = u_{TGO} - u_{in} \tag{15}$$

according to the third step of MTB. Hence, the scattered GO field can be expressed as

$$u_{SGO}(P) = R_1 u_r U\left(-\xi_{r+}\right) - R_2 u_0 e^{jk\rho\cos(\phi+\phi_0)} U\left(-\xi_{0+}\right) \tag{16}$$

according to (15).

D. DIFFRACTED WAVE

There are two transition boundaries in our problem. One of them is f_1, which arises from the anomalously reflecting metasurface and the other one is f_2, which comes from the impedance surface. The transition boundaries of f_1 and f_2 can be defined by

$$f_1\left(\pi-\phi_r,\phi_r\right) = -\sin\phi_r R_1 e^{jkx_s(\cos\phi_0 - \cos\phi_r)} \tag{17}$$

and

$$f_2\left(\pi-\phi_0,\phi_0\right) = \sin\phi_0 R_2 \tag{18}$$

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according to (4). After that, the functions of f_1 and f_2 can be arranged by considering the fourth step of MTB.

First, the function of f_1 can be obtained step by step as

$$f_1(\pi-\phi_r,\phi_r)=-\sin\phi_r R_1 e^{jkx_s(\cos\phi_0-\cos\phi_r)}, \quad (19)$$

which can also be written as

$$f_1(\pi-\phi_r,\phi_r)=-\sin\phi_r\frac{\sin\phi_0-\sin\theta_1}{\sin\phi_r+\sin\theta_1}e^{jkx_s(\cos\phi_0-\cos\phi_r)}, \quad (20)$$

which can be arranged as

$$f_1(\pi-\phi_r,\phi_r)=-\left(\frac{\sin\phi_0}{\sin\theta_1}\frac{\sin\phi_r\sin\theta_1}{\sin\phi_r+\sin\theta_1}+\frac{\sin\phi_r\sin\theta_1}{\sin\phi_r+\sin\theta_1}\right)e^{jkx_s(\cos\phi_0-\cos\phi_r)} \quad (21)$$

where the term of split function K_+ can be expressed by the relation of

$$K_+(\pi-\phi_r,\theta_1)K_+(\phi_r,\theta_1)=\frac{\sin\phi_r\sin\theta_1}{\sin\phi_r+\sin\theta_1}. \quad (22)$$

As a result, the function f_1 can be written as

$$f_1(\pi-\phi_r,\phi_r)=\left(1-\frac{\sin\phi_0}{\sin\theta_1}\right)K_+(\pi-\phi_r,\theta_1)\times K_+(\phi_r,\theta_1)e^{jkx_s(\cos\phi_0-\cos\phi_r)} \quad (23)$$

by considering the fourth step of MTB.

Secondly, the function f_2 can be found step by step as

$$f_2(\pi-\phi_0,\phi_0)=\sin\phi_0 R_2, \quad (24)$$

which can also be expressed as

$$f_2(\pi-\phi_0,\phi_0)=\sin\phi_0\frac{\sin\phi_0-\sin\theta_2}{\sin\phi_0+\sin\theta_2}, \quad (25)$$

which can be simplified as

$$f_2(\pi-\phi_0,\phi_0)=\left(\frac{\sin\phi_0}{\sin\theta_2}-1\right)K_+(\pi-\phi_0,\theta_2)K_+(\phi_0,\theta_2) \quad (26)$$

by considering the fourth step of MTB.

According to the fifth and final step of the MTB, the reciprocity theorem must be satisfied by the functions of $f_1(\phi,\phi_r)$ and $f_2(\phi,\phi_0)$. The functions of $f_1(\phi,\phi_r)$ and $f_2(\phi,\phi_0)$ can be obtained by using the relation of $\phi=\pi-\phi_r$ and $\phi=\pi-\phi_0$ in (23) and (26), respectively.

The function $f_1(\phi,\phi_r)$ which can be expressed by considering (23) can be arranged for satisfying the reciprocity theorem as

$$f_1(\phi,\phi_r)=\left[1-\frac{2\cos\left(\frac{\phi-\phi_0+\phi_r}{2}\right)\cos\frac{\phi_0}{2}}{\sin\theta_1}\right]\times K_+(\phi,\theta_1)K_+(\phi_r,\theta_1)e^{jkx_s(\cos\phi_0-\cos\phi_r)} \quad (27)$$

for the anomalously reflecting metasurface effect to the diffraction according to the fifth step of MTB.

The function $f_2(\phi,\phi_0)$ which can be expressed by considering (26) can be arranged for satisfying the reciprocity theorem as

$$f_2(\phi,\phi_0)=\left[\frac{2\sin\frac{\phi}{2}\sin\frac{\phi_0}{2}}{\sin\theta_2}-1\right]K_+(\pi-\phi,\theta_2)K_+(\pi-\phi_0,\theta_2) \quad (28)$$

for the impedance half-plane effect to the diffraction according to the fifth step of MTB. The terms of $\pi-\phi$ and $\pi-\phi_0$ are written for taking the impedance half-plane from the right side to the left side as in the geometry in Fig. 1.

Finally, the total nonuniform diffracted wave expression u_d can be written as

$$u_d=u_{d1}+u_{d2} \quad (29)$$

where u_{d1}, which is the diffracted wave expression coming from the anomalously reflecting metasurface can be expressed as

$$u_{d1}=u_0\frac{e^{-j\frac{\pi}{4}}}{\sqrt{2\pi}}\left[1-\frac{2\cos\left(\frac{\phi-\phi_0+\phi_r}{2}\right)\cos\frac{\phi_0}{2}}{\sin\theta_1}\right]\times K_+(\phi,\theta_1)K_+(\phi_r,\theta_1)e^{jkx_s(\cos\phi_0-\cos\phi_r)}\frac{e^{-jk\rho}}{\sqrt{k\rho}} \quad (30)$$

and u_{d2}, which is the diffracted wave expression obtained from the impedance half-plane can be defined as

$$u_{d2}=u_0\frac{e^{-j\frac{\pi}{4}}}{\sqrt{2\pi}}\left[\frac{2\sin\frac{\phi}{2}\sin\frac{\phi_0}{2}}{\sin\theta_2}-1\right]\times K_+(\pi-\phi,\theta_2)K_+(\pi-\phi_0,\theta_2)\frac{e^{-jk\rho}}{\sqrt{k\rho}} \quad (31)$$

for the nonuniform diffracted wave.

E. UNIFORM DIFFRACTED WAVE

In order to obtain the uniform field expressions, the uniform form of the asymptotic relation of

$$p_s(\phi,\alpha)\approx\frac{e^{j\frac{\pi}{4}}}{\sqrt{2\pi}}\frac{2\sin\frac{\phi}{2}\sin\frac{\alpha}{2}}{\cos\phi+\cos\alpha}\frac{e^{-jk\rho}}{\sqrt{k\rho}} \quad (32)$$

can be used [29]. The uniform form of the relation in (32) can be given as

$$p_s(\phi,\alpha)=e^{jk\rho\cos(\phi-\alpha)}sign\left(-\sqrt{2k\rho}\cos\frac{\phi-\alpha}{2}\right)\times F\left[\left|-\sqrt{2k\rho}\cos\frac{\phi-\alpha}{2}\right|\right]-e^{jk\rho\cos(\phi-\alpha)}sign\left(-\sqrt{2k\rho}\cos\frac{\phi+\alpha}{2}\right)$$

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$$\times F\left[\left|-\sqrt{2k\rho}\cos\frac{\phi+\alpha}{2}\right|\right] \tag{33}$$

where the $sign(x)$ and $F[x]$ are the sign and Fresnel functions, respectively.

After that, the uniform field expression u_d^u can be written as

$$u_d^u = u_{d1}^u + u_{d2}^u \tag{34}$$

where u_{d1}^u, which is the uniform diffracted wave expression coming from the anomalously reflecting metasurface can be expressed as

$$u_{d1}^u = u_0\left[1-\frac{2\cos\left(\frac{\phi-\phi_0+\phi_r}{2}\right)\cos\frac{\phi_0}{2}}{\sin\theta_1}\right] \times\frac{K_+(\phi,\theta_1)K_+(\phi_r,\theta_1)}{2\sin\frac{\phi}{2}\sin\frac{\phi_r}{2}}e^{jkx_s(\cos\phi_0-\cos\phi_r)}p_s(\phi,\phi_r) \tag{35}$$

and u_{d2}^u, which is the diffracted wave expression obtained from the impedance half-plane can be given as

$$u_{d2}^u = u_0\left[\frac{2\sin\frac{\phi}{2}\sin\frac{\phi_0}{2}}{\sin\theta_2}-1\right] \times\frac{K_+(\pi-\phi,\theta_2)K_+(\pi-\phi_0,\theta_2)}{2\cos\frac{\phi}{2}\cos\frac{\phi_0}{2}}p_s(\pi-\phi,\pi-\phi_0) \tag{36}$$

for the uniform diffracted wave.

IV. PHYSICAL OPTICS SOLUTION

In this section, we will offer a physical optics (PO) solution for the problem. We will compare and verify the results of MTB solution of the problem with the well-known theory of PO solution. The general scattered field definition of the PO expression can be given by

$$u_s = \frac{ke^{j\frac{\pi}{4}}}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(\beta,\phi)\left(\sin\frac{\beta-\phi_0}{2}-\sin\frac{\beta+\phi_0}{2}\right) \times\frac{e^{jk(x'\cos\phi_0-R)}}{\sqrt{kR}}dx' \tag{37}$$

for the two-dimensional plane problems. β is a variable angle being a function of x'. R which is the distance between the integration and observation points can be expressed by

$$R=\sqrt{\rho^2+(x')^2-2\rho x'\cos\phi}. \tag{38}$$

f is a function that is equal to

$$f(\beta,\phi) = -\frac{\sin\frac{\beta+\phi_0}{2}-\sin\theta}{\sin\frac{\beta+\phi_0}{2}+\sin\theta}, \tag{39}$$

for an impedance half-plane problem [26], [49]. The PO definition of total scattered field of the problem can also be represented by

$$u_{sT}^{PO} = u_{s1}^{PO} + u_{s2}^{PO}, \tag{40}$$

where u_{s1}^{PO} is the scattered field arising from the anomalously reflecting metasurface that is equal to

$$u_{s1}^{PO} = \frac{ke^{j\frac{\pi}{4}}}{\sqrt{2\pi}}\int_0^{\infty}\sin\frac{\beta+\phi_r}{2}e^{jkx_s(\cos\phi_0-\cos\phi_r)} \times\frac{\sin\frac{\beta+\phi_0}{2}-\sin\theta_1}{\sin\frac{\beta+\phi_r}{2}+\sin\theta_1}\frac{e^{jk(x_1'\cos\phi_r-R)}}{\sqrt{kR}}dx_1' \tag{41}$$

and u_{s2}^{PO} is the scattered field originating from the impedance surface that is equal to

$$u_{s2}^{PO} = \frac{ke^{j\frac{\pi}{4}}}{\sqrt{2\pi}}\int_{-\infty}^{0}\sin\frac{\beta+\phi_0}{2}\frac{\sin\frac{\beta+\phi_0}{2}-\sin\theta_2}{\sin\frac{\beta+\phi_0}{2}+\sin\theta_2} \times\frac{e^{jk(x_2'\cos\phi_0-R)}}{\sqrt{kR}}dx_2'. \tag{42}$$

As a result, the total field expression of the PO solution that can be obtained by the sum of the total scattered PO field in (40) and the incident field in (1) can be represented by

$$u_T^{PO} = u_i + u_{s1}^{PO} + u_{s1}^{PO}. \tag{43}$$

V. NUMERICAL RESULTS

In this section, we will analyze the behaviors of the total field and its components numerically. The distance of observation is taken as 3λ where λ is the wavelength. The angle of observation (ϕ) changes from 0° to 180° since the junction combination of two planar planes is located at $x\in(-\infty,\infty)$. The angle of incidence (ϕ_0) is equal to 60°. The different values will be taken into consideration for θ_1, θ_2 and ϕ_r.

Fig. 3 shows the variation of the total wave versus the observation angle for ϕ_r, $\sin\theta_1$ and $\sin\theta_2$ which are equal to 30°, 2 and 3, respectively. The values of $\sin\theta_1$ and $\sin\theta_2$ are chosen as the arbitrary values that depicts having the different impedance values of sheets relative to each other, as in [48]. Until 150°, the diffracted, reflected GO and incident GO waves interfere with each other. After this value, which is the reflection boundary, only the incident GO and diffracted waves exist.

It can be seen from Fig. 4 that there are interferences between two surfaces of anomalously reflecting

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metasurface and impedance half-plane. The reflection boundaries are also read in the degrees of 120° and 150° which are the degrees of reflection boundaries of impedance half-plane and anomalously reflecting metasurface, respectively. The behavior of the total GO and diffracted waves can be observed clearly.

Fig. 5 depicts the variation of the total wave concerning the observation angle for different values of $\sin\theta_1$ and $\sin\theta_2$. The angle of reflection is 30°. The main difference is observed in the intensity of the reflected GO wave and accompanies with ϕ is smaller than the reflection boundary of 150°. After the reflection boundary, only the diffracted waves are the cause of the difference between the intensities.

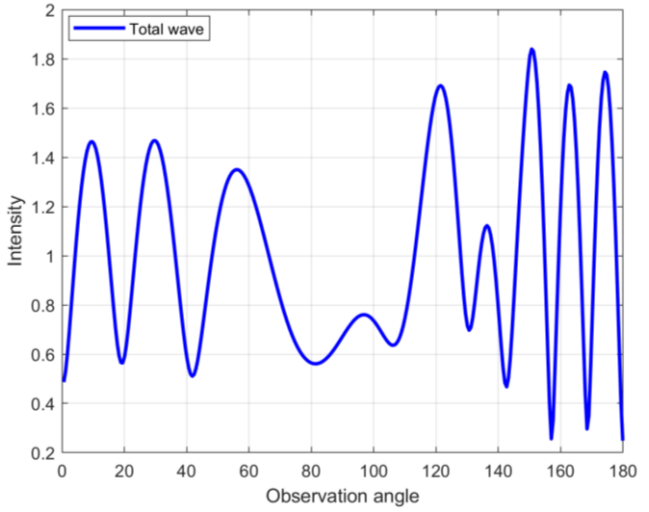

FIGURE 3. Total wave for $\phi_r = 30°$, $\sin\theta_1 = 2$ and $\sin\theta_2 = 3$.

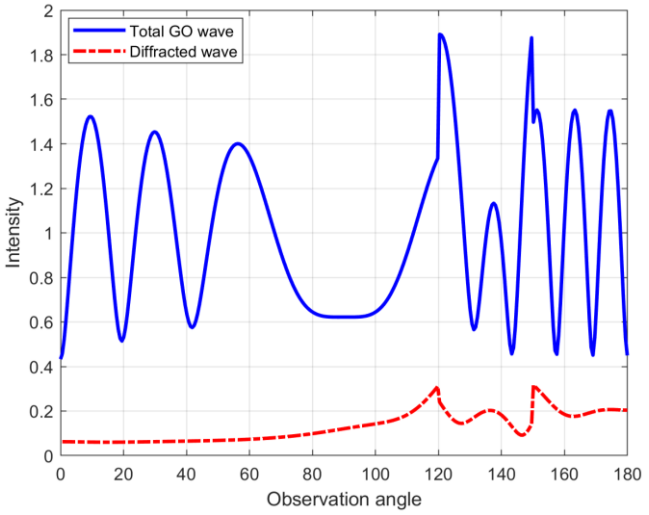

FIGURE 4. Total GO and diffracted waves for $\phi_r = 30°$, $\sin\theta_1 = 2$ and $\sin\theta_2 = 3$.

Fig. 6 shows the variation of the total wave for different values of ϕ_r. A phase difference is seen before 150°. The difference between the reflection angles also affects the intensity of the total wave. The two total waves are in harmony after 150°.

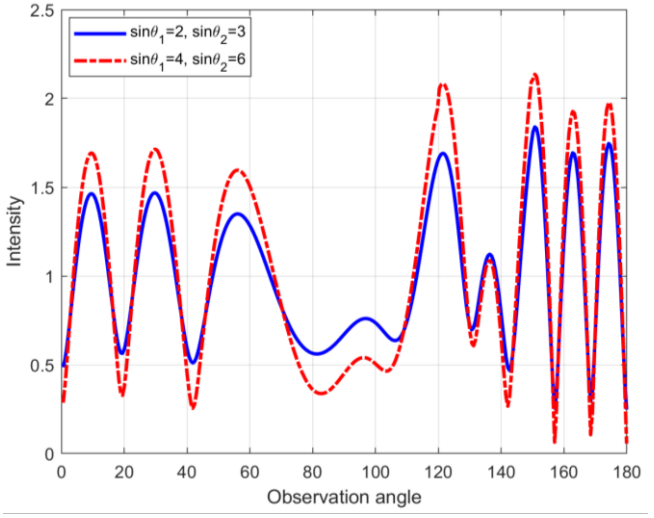

FIGURE 5. Total wave for different values of $\sin\theta_1$ and $\sin\theta_2$.

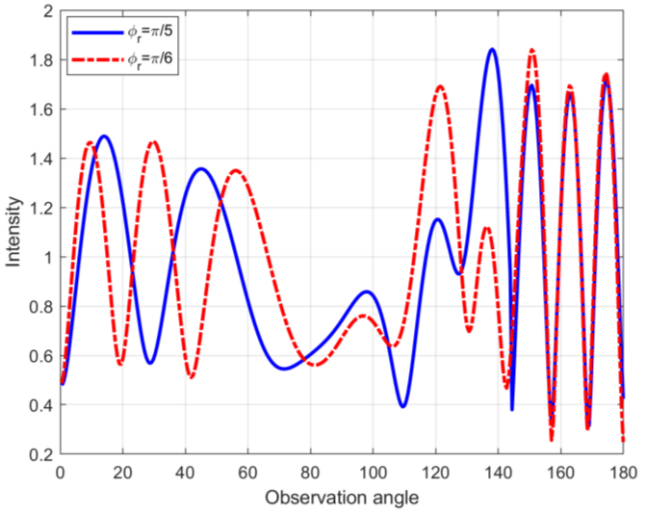

FIGURE 6. Total wave for different values of ϕ_r.

In Fig. 7(a) and Fig. 7(b), the comparison of two solutions, obtained by the techniques of MTB in Section III and PO in Section IV, is given for different values of $\sin\theta_1$ and $\sin\theta_2$. It can be observed from the figures that the behavior of the two total waves are similar for different values of $\sin\theta$.

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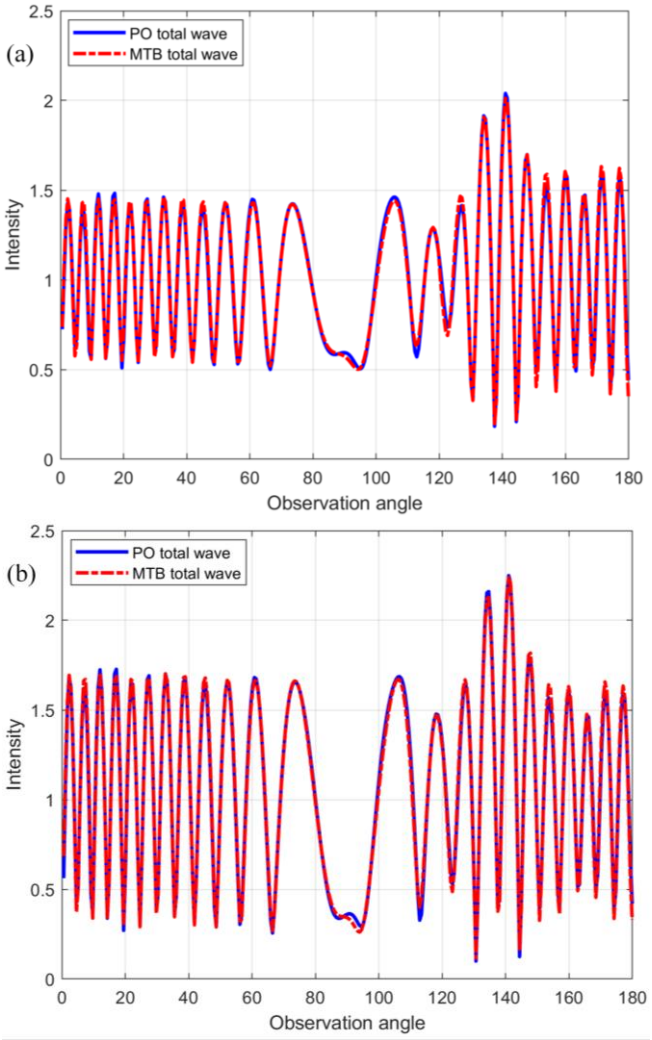

FIGURE 7. **Comparison of the total wave results of MTB and PO for $\phi_r = 30°$. The distance of observation is 6λ. (a) $\sin\theta_1 = 2$ and $\sin\theta_2 = 3$. (b) $\sin\theta_1 = 4$ and $\sin\theta_2 = 3$.**

VI. CONCLUSIONS

In this paper, we investigated the scattering problem of waves by a planar junction between anomalously reflecting metasurface and impedance half-planes. The effect of the phase shift and the amplitude variation of the anomalously reflecting metasurface is taken into consideration during the evaluation of the expressions. The expressions of the total, total GO and diffracted waves are obtained by using the novel technique of MTB and the behaviors of the waves are also studied numerically. The result of MTB is also compared and verified by the well-known theory of PO. The comparison shows that the total field result of MTB is also in agreement with the total field result of PO.

In these types of problems, the surface waves can also be obtained [50], [51]. After the scattered wave result is obtained by the technique of MTB, the scattered wave expression of the MTB is first transformed into the complex integral as in the several diffraction problems applied in [34], then the surface waves can be obtained by the saddle point evaluation of the complex integral. This study can also be considered in a future work paper.

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